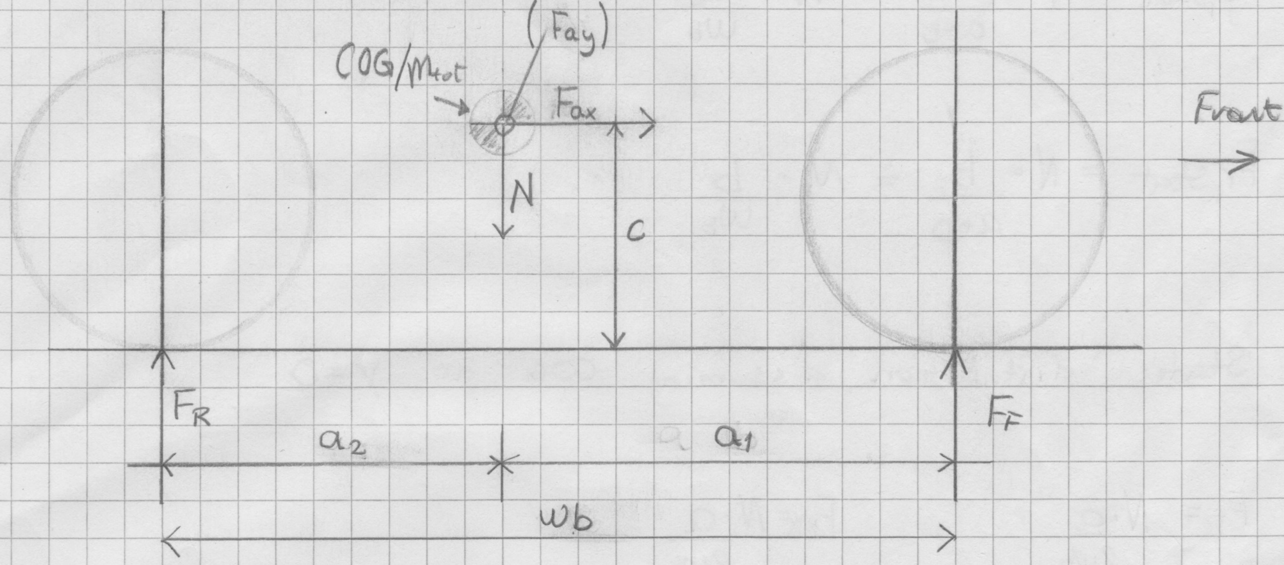
Problem 1 a

Weight Shift:

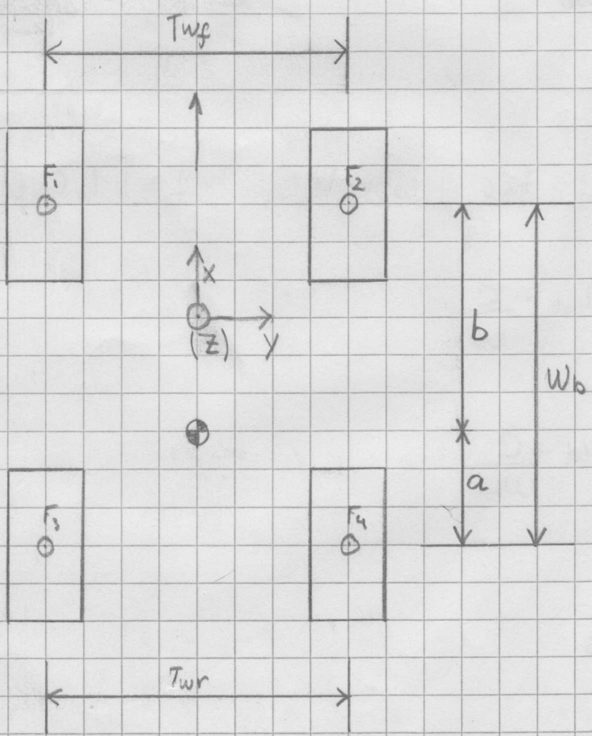


$N = m_{tot} \cdot g$

m_{tot}

Acceleration vector

$A = [a_x, a_y, a_z]$



Static weight distribution:

$$F_{f,stat} = N \cdot \frac{a}{a+b} = N \cdot \frac{a}{wb}$$

$$F_{r,stat} = N \cdot \frac{b}{a+b} = N \cdot \frac{b}{wb}$$

Static distribution, assuming COG at $y=0$

$$F_{1,s} = N \cdot \frac{a}{2wb} \qquad F_{2,s} = N \cdot \frac{a}{2wb}$$

$$F_{3,s} = N \cdot \frac{b}{2wb} \qquad F_{4,s} = N \frac{b}{2wb}$$

Dynamic weight shifting two wheels $A = [1,0,0]$

$$F_{f,dyn} = F_{ax} \cdot \frac{c}{a+b} = a_x \cdot m_{tot} \cdot \frac{c}{wb}$$

$$F_{r,dyn} = F_{ax} \cdot \frac{c \cdot (-1)}{a+b} = -a_x \cdot m_{tot} \cdot \frac{c}{wb}$$

$A = [0,1,0]$ Two wheels

$$F_{rs,dyn} = F_{ay} \cdot \frac{C}{Tw} = a_y \cdot m_{tot} \cdot \frac{C}{Tw}$$

$$F_{Ls,dyn} = F_{ay} \cdot \frac{C \cdot (-1)}{Tw} = -a_y \cdot m_{tot} \cdot \frac{C}{Tw}$$

Dynamic distribution

$$F_{1,d} = F_{ay} \cdot \frac{C}{2T_{wf}} + F_{ax} \cdot \frac{C}{2W_b} \qquad F_{2,d} = -F_{ay} \cdot \frac{C}{T_{wf}} + F_{ax} \cdot \frac{C}{2W_b}$$

$$F_3 = F_{ay} \cdot \frac{C}{2T_{wr}} - F_{ax} \cdot \frac{C}{2W_b} \qquad F_{3,d} = -F_{ay} \cdot \frac{C}{T_{wr}} - F_{ax} \cdot \frac{C}{2W_b}$$

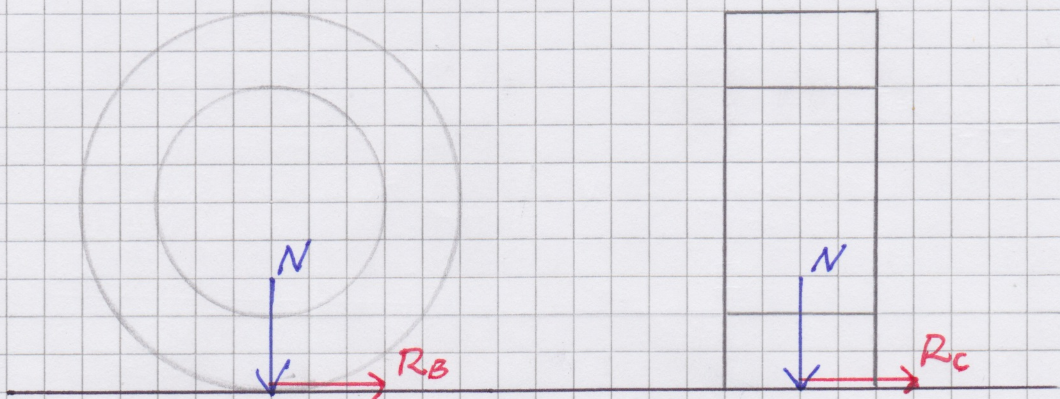
Total distribution

$$F_1 = \frac{N \cdot a}{2W_b} + \frac{C}{2}\left(\frac{F_{ay}}{T_{wf}} + \frac{F_{ax}}{W_b} \right) \qquad F_2 = \frac{N \cdot a}{2W_b} + \frac{C}{2}\left(-\frac{F_{ay}}{T_{wf}} + \frac{F_{ax}}{W_b} \right)$$

$$F_3 = \frac{N \cdot b}{2W_b} + \frac{C}{2}\left(\frac{F_{ay}}{T_{wr}} - \frac{F_{ax}}{W_b} \right) \qquad F_4 = \frac{N \cdot b}{2W_b} + \frac{C}{2}\left(-\frac{F_{ay}}{T_{wr}} - \frac{F_{ax}}{W_b} \right)$$

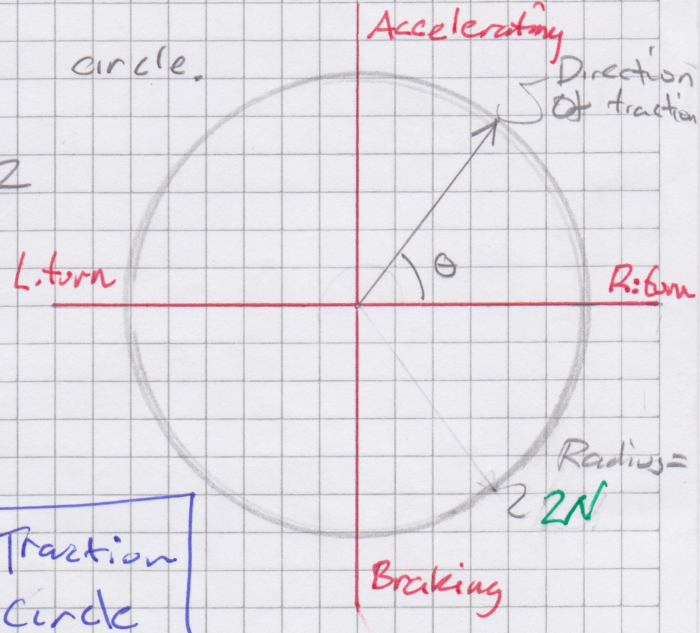
Remember wing downforce

Problem 1 b



Assume circular traction circle.

Assume friction coeff $\mu = 2$
⇒ Radius of traction circle is $N \cdot \mu = 2N$



⇒ Corner/Braking forces follow this rule:

$2N \leq \sqrt{R_B^2 + R_c^2}$

$\boxed{\text{Traction circle}}$

This means tire can provide traction (forces) in any direction by a factor of 2 times the tire load force

N = Tire load force

R_c = Cornering forces

R_B = Braking/accelerating forces

Problem 1b

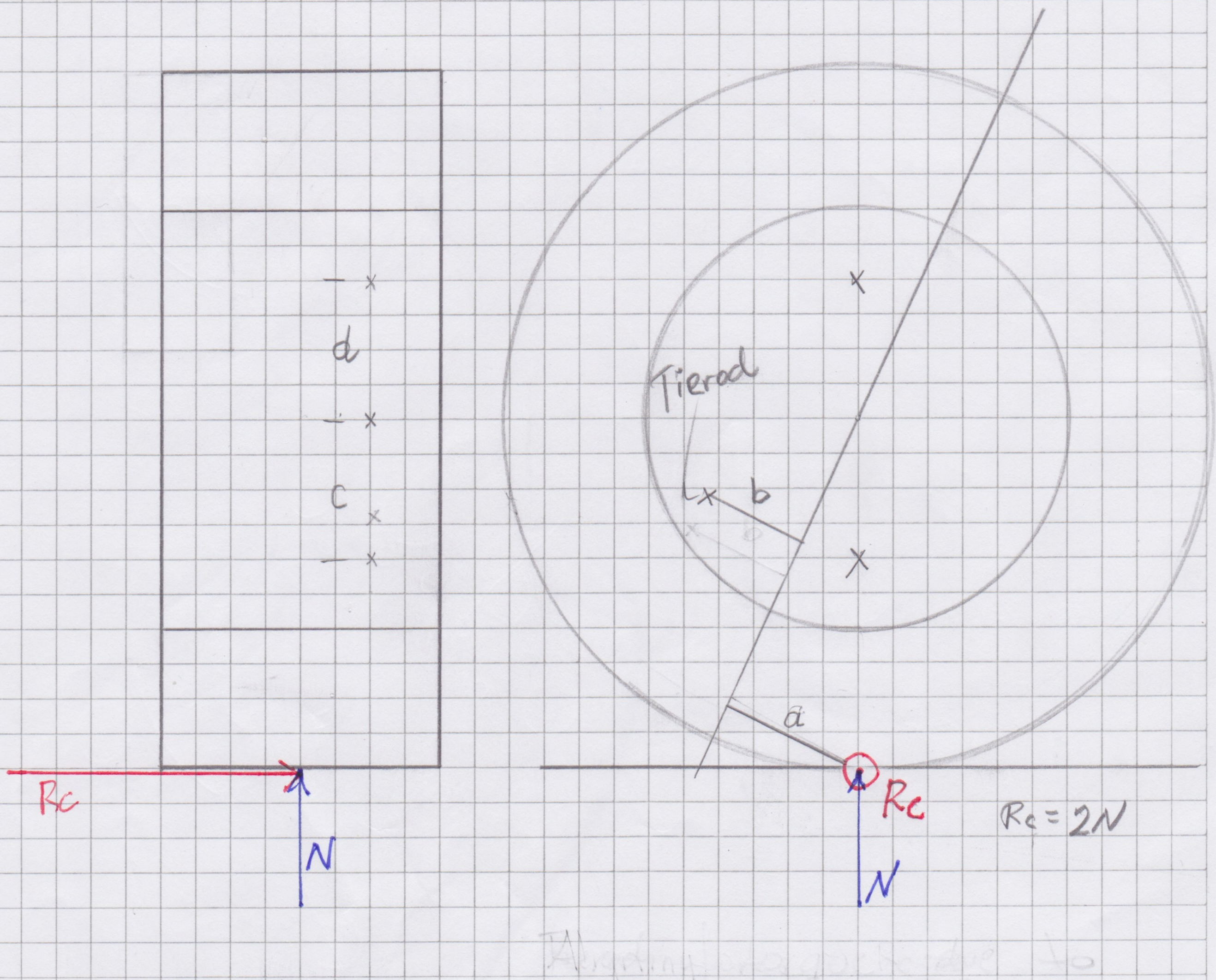
Mechanical trail → Aligning torque

Scrub radius → Aligning torque

Cornering forces → Overturning Moment

Friction/traction force centre applies with an offset to the kingpin axis

Cornering case



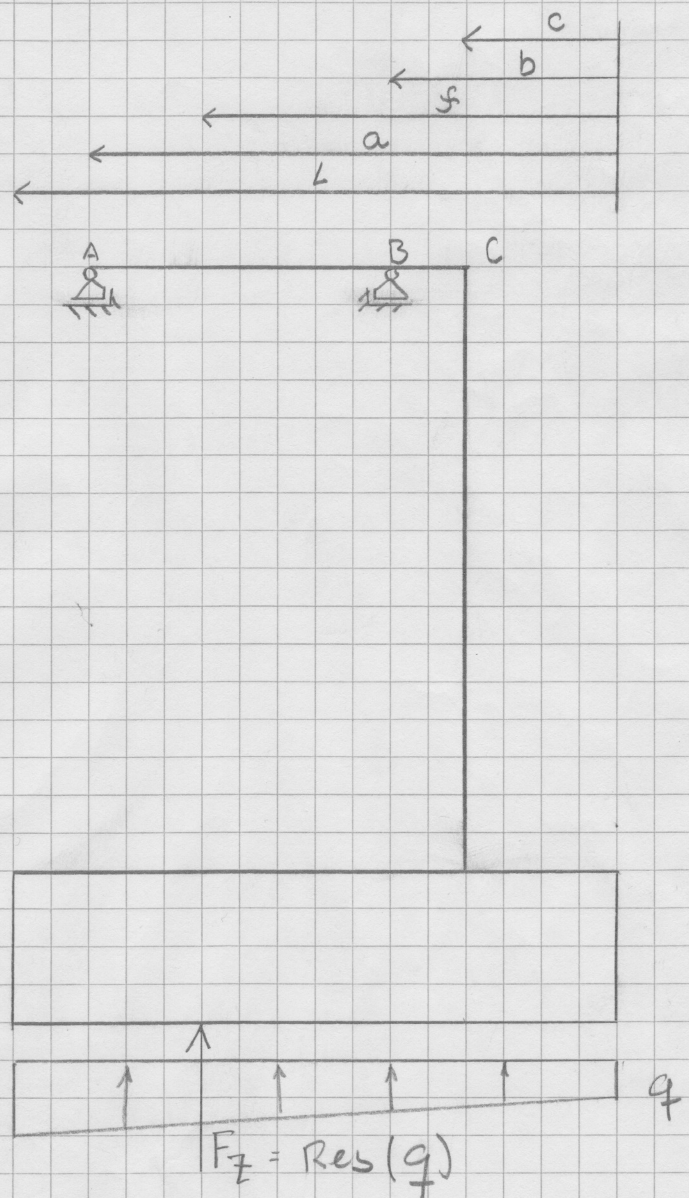
Normal force on tire:

$\sum M_A = 0 \qquad F_z \cdot (a-f) + B_z(a-b) = 0 \qquad B_z = -F_z \dfrac{(a-f)}{(a-b)}$

$\sum M_B = 0 \qquad F_z \cdot (b-f) + A_z(b-a) = 0 \qquad A_z = -F_z \dfrac{(b-f)}{(b-a)}$

$$A = \left[0,\ 0,\ -F_z \frac{(b-f)}{(b-a)} \right]$$

$$B = \left[0,\ 0,\ -F_z \frac{(a-f)}{(a-b)} \right]$$



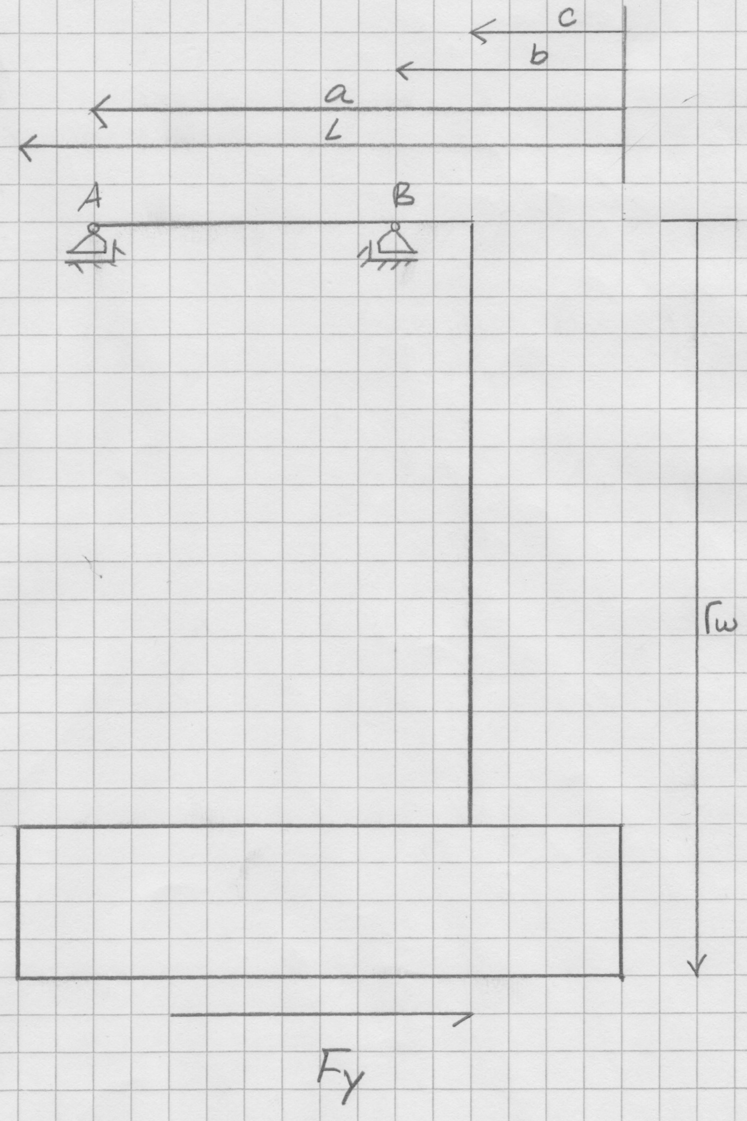
Cornering forces:

$\sum M_A = 0 \qquad F_y \cdot r_w + B_z(a-b) = 0 \qquad B_z = -F_y \cdot \frac{r_w}{a-b}$

$\sum M_B = 0 \qquad F_y \cdot r_w - A_z(a-b) = 0 \qquad A_z = F_y \cdot \frac{r_w}{a-b}$

$A = \begin{bmatrix} 0 & 0 & F_y \cdot \frac{r_w}{a-b} \end{bmatrix}$

$B = \begin{bmatrix} 0 & 0 & -F_y \cdot \frac{r_w}{a-b} \end{bmatrix}$



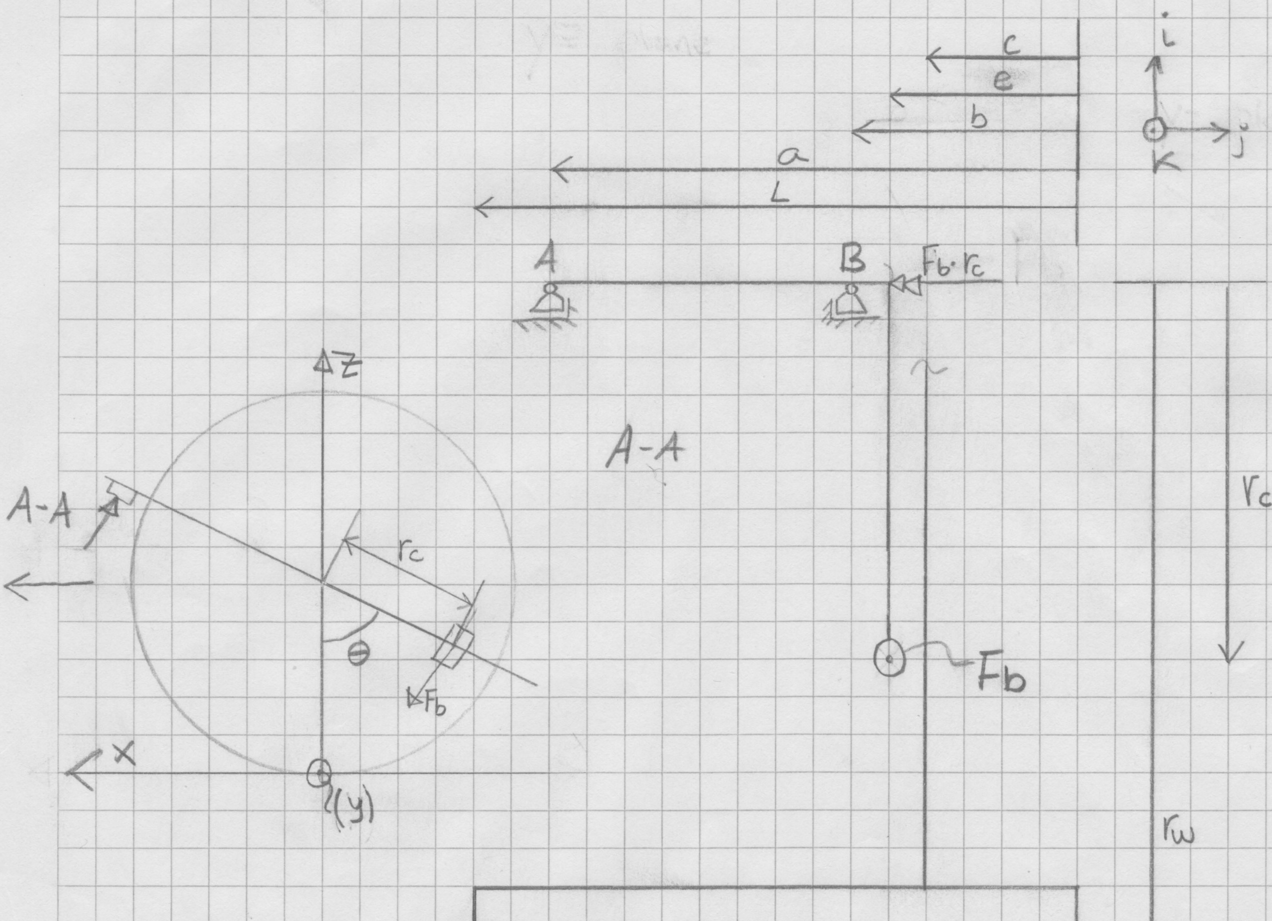
Caliper braking forces on tire

$$\sum M_A = 0 \quad Fb\cdot(a-e) + Bk\cdot(a-b) \qquad Bk = -Fb\cdot\frac{(a-e)}{(a-b)}$$

$$\sum M_B = 0 \quad Fb\cdot(b-e) - Ak\,(a-b) \qquad Ak = Fb\cdot\frac{(b-e)}{(a-b)}$$

$$A = \left[\underset{A_x}{\cos\theta\cdot Fb\cdot\frac{(b-e)}{(a-b)}},\ \underset{A_y}{0},\ \underset{A_z}{-\sin\theta\cdot Fb\frac{(b-e)}{(a-b)}}\right]$$

$$B = \left[\underset{B_x}{-\cos\theta\cdot Fb\,\frac{(a-e)}{(a-b)}},\ \underset{B_y}{0},\ \underset{B_z}{\sin\theta\cdot\frac{(a-e)}{(a-b)}}\right]$$



Braking forces on tire

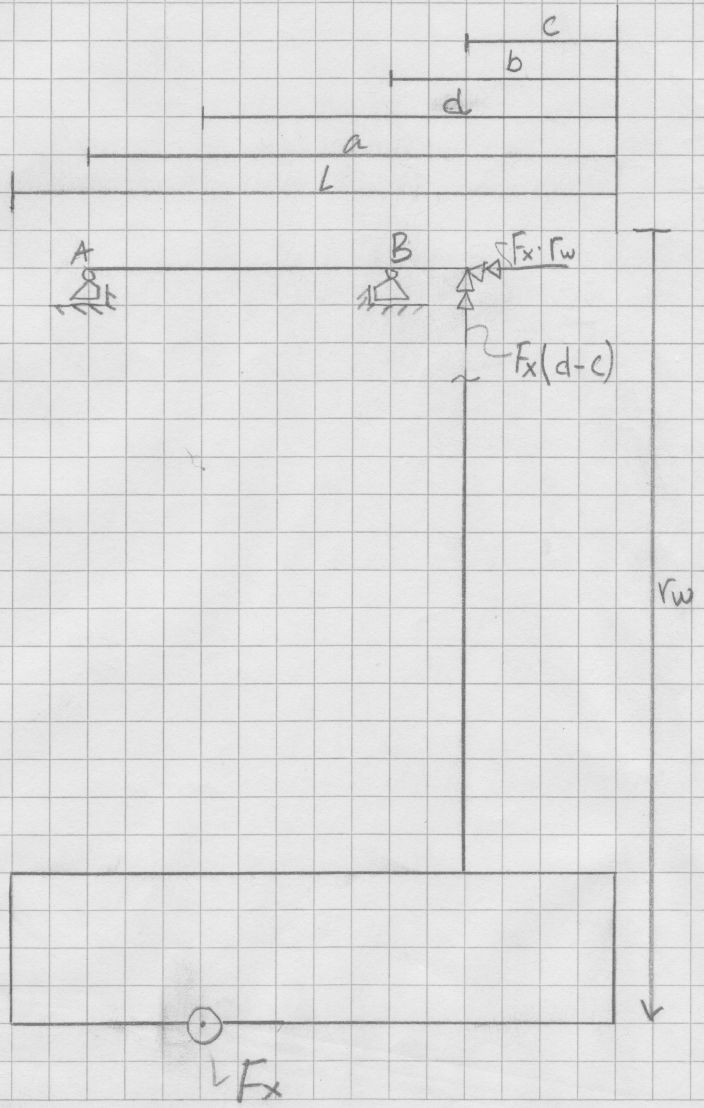
$T_V = [F_x, 0, 0]$ Pk. 4

$\sum M_A = 0 \quad F_x \cdot (a-d) + F_{Bx} \cdot (a-b) = 0 \quad F_{Bx} = -F_x \cdot \frac{(a-d)}{(a-b)}$

$\sum M_B = 0 \quad F_x \cdot (d-b) + F_{Ax}(a-b) = 0 \quad F_{Ax} = -F_x \cdot \frac{(d-b)}{(a-b)}$

$$A = \left[-F_x \cdot \frac{(d-b)}{(a-b)}, 0, 0 \right]$$

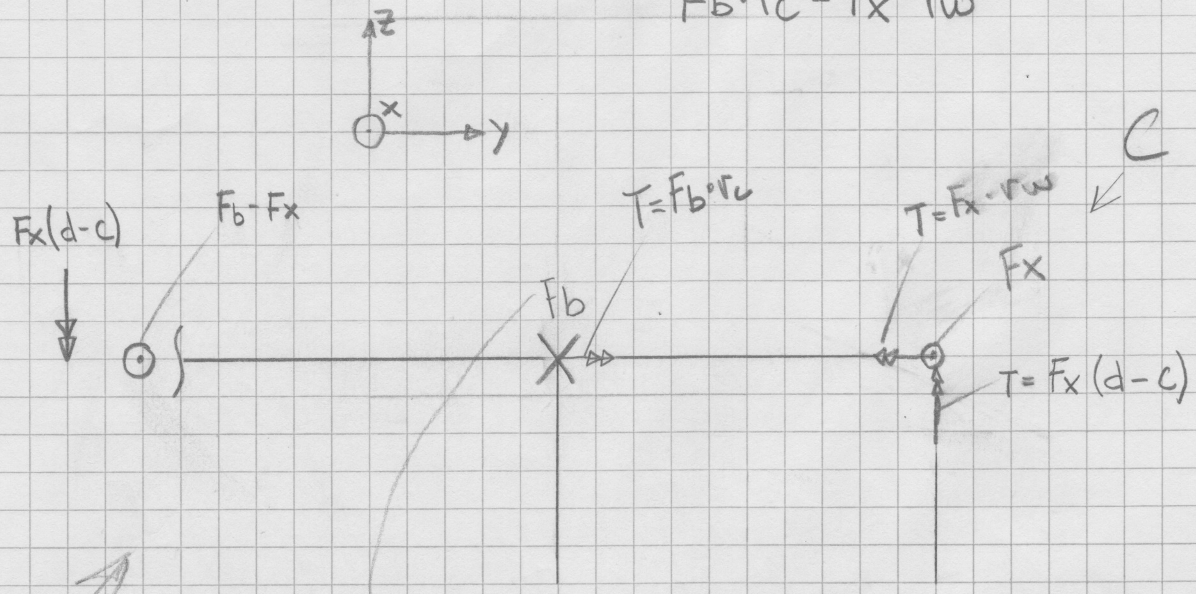
$$B = \left[-F_x \cdot \frac{(a-d)}{(a-b)}, 0, 0 \right]$$



Control

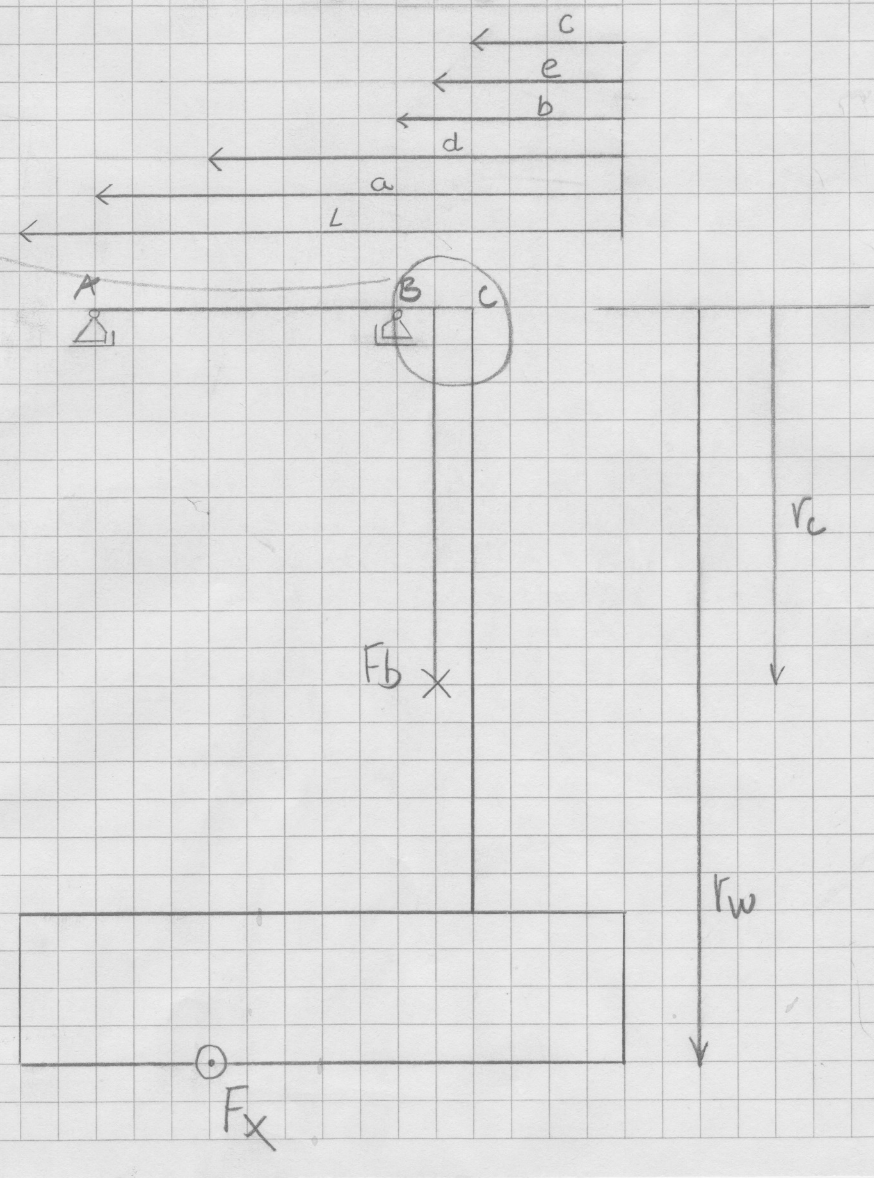
P1C5

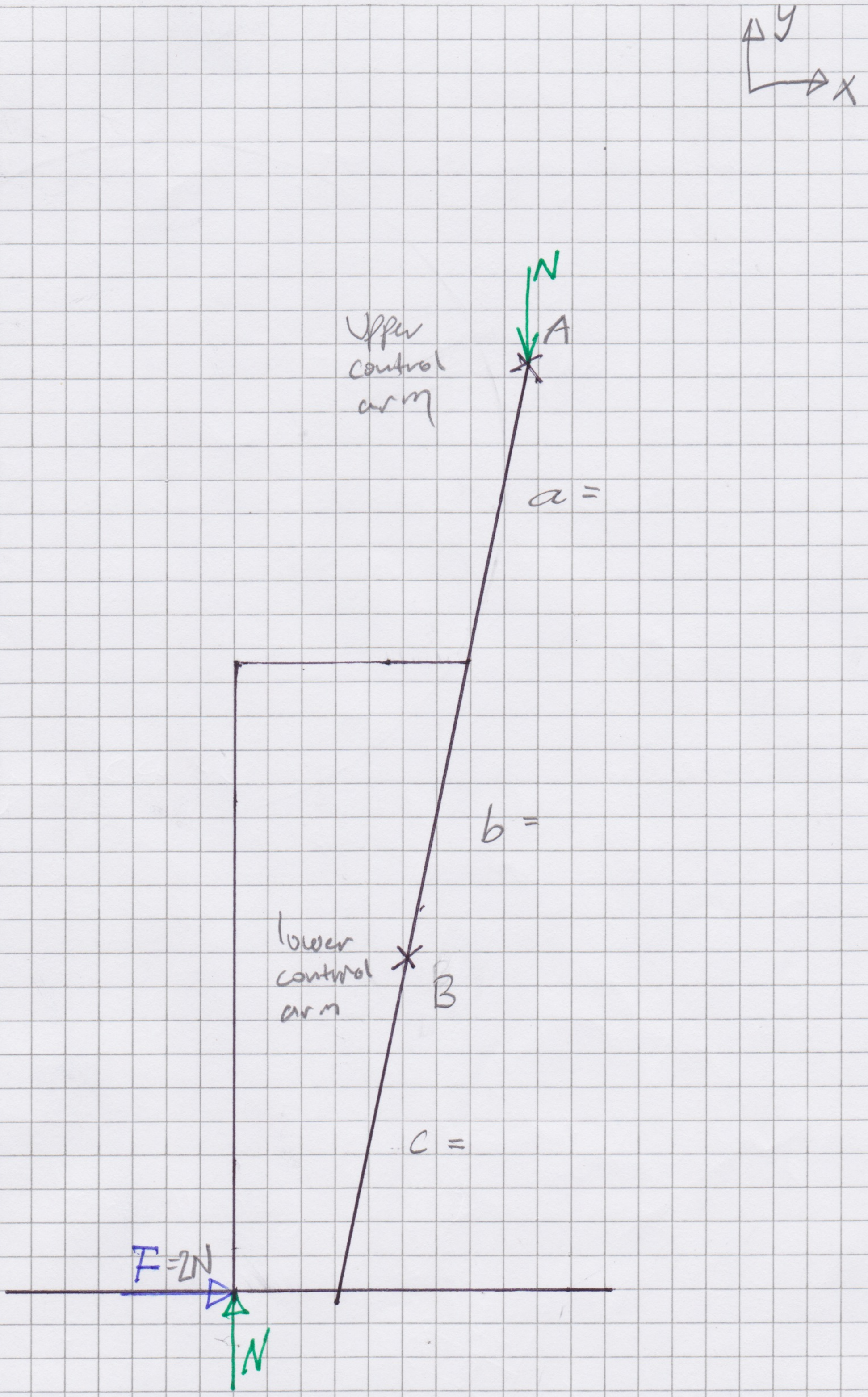
$$F_b \cdot r_c = F_x \cdot r_w$$



$$F_{bx} = F_b \cdot \cos(\theta)$$

$$F_{bz} = F_b \cdot \sin(\theta)$$



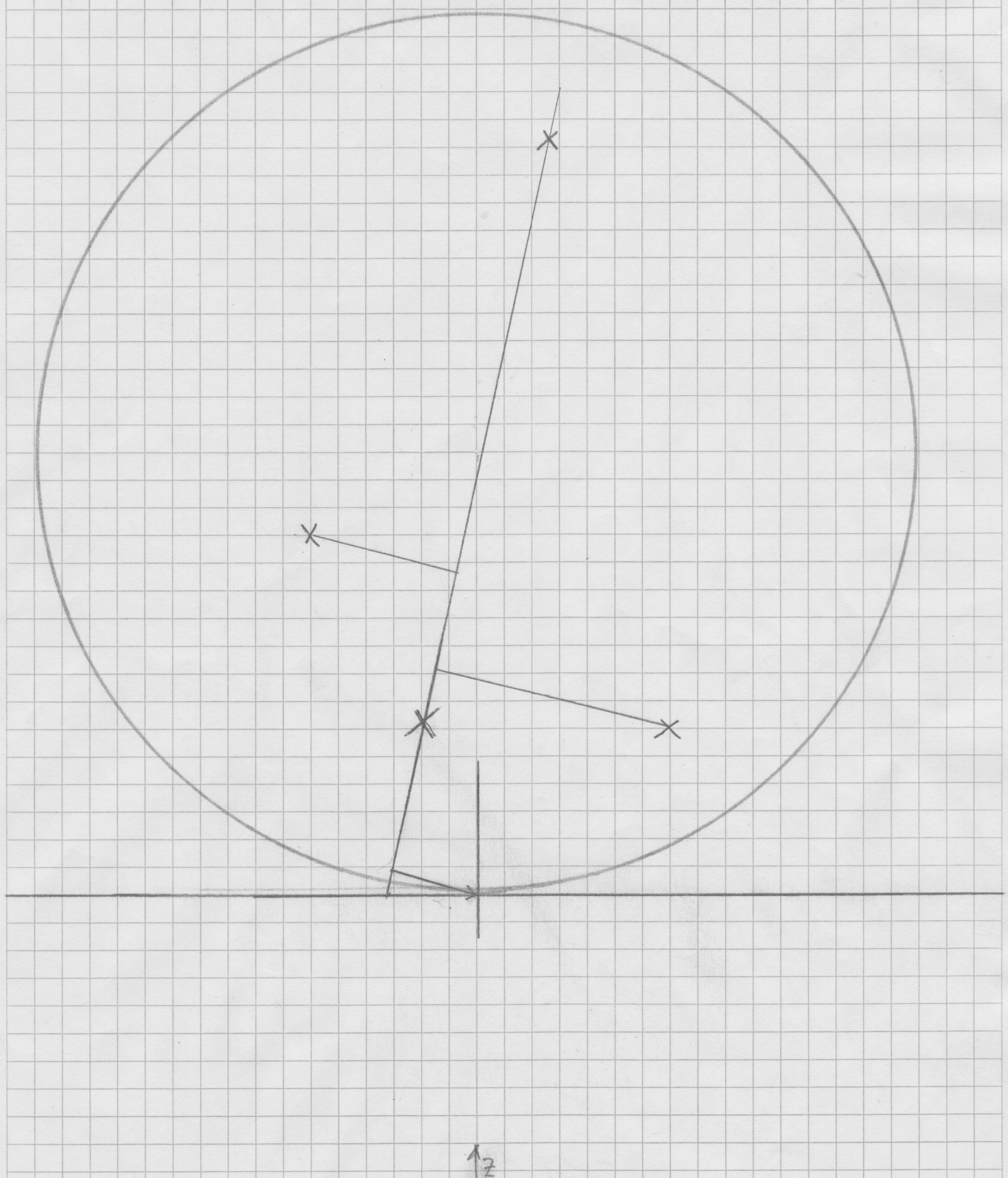


$$F_{Ax} = F \cdot \frac{c}{a+b}$$

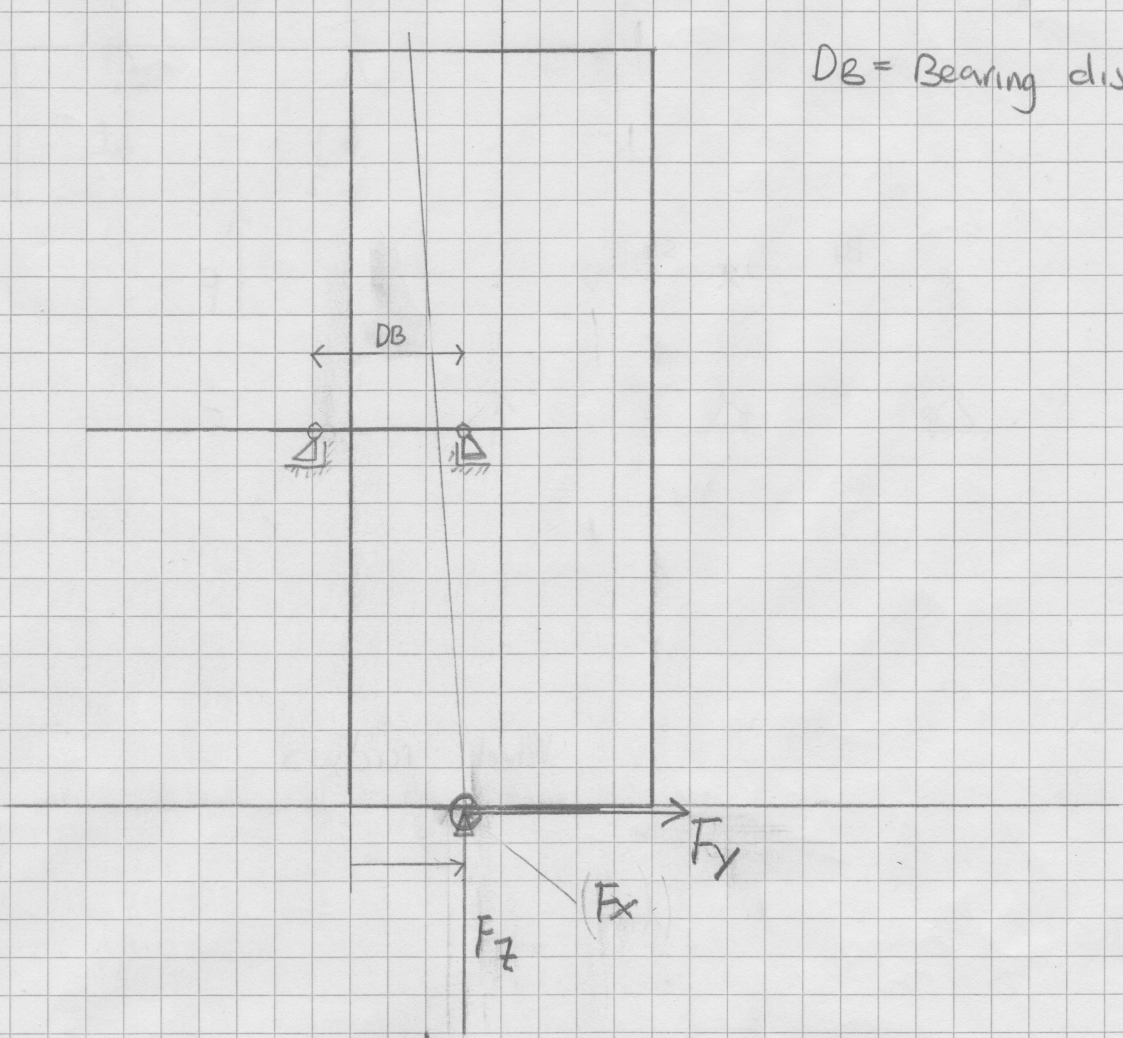
$$F_{Bx} = F \cdot \frac{a+b+c}{a+b}$$

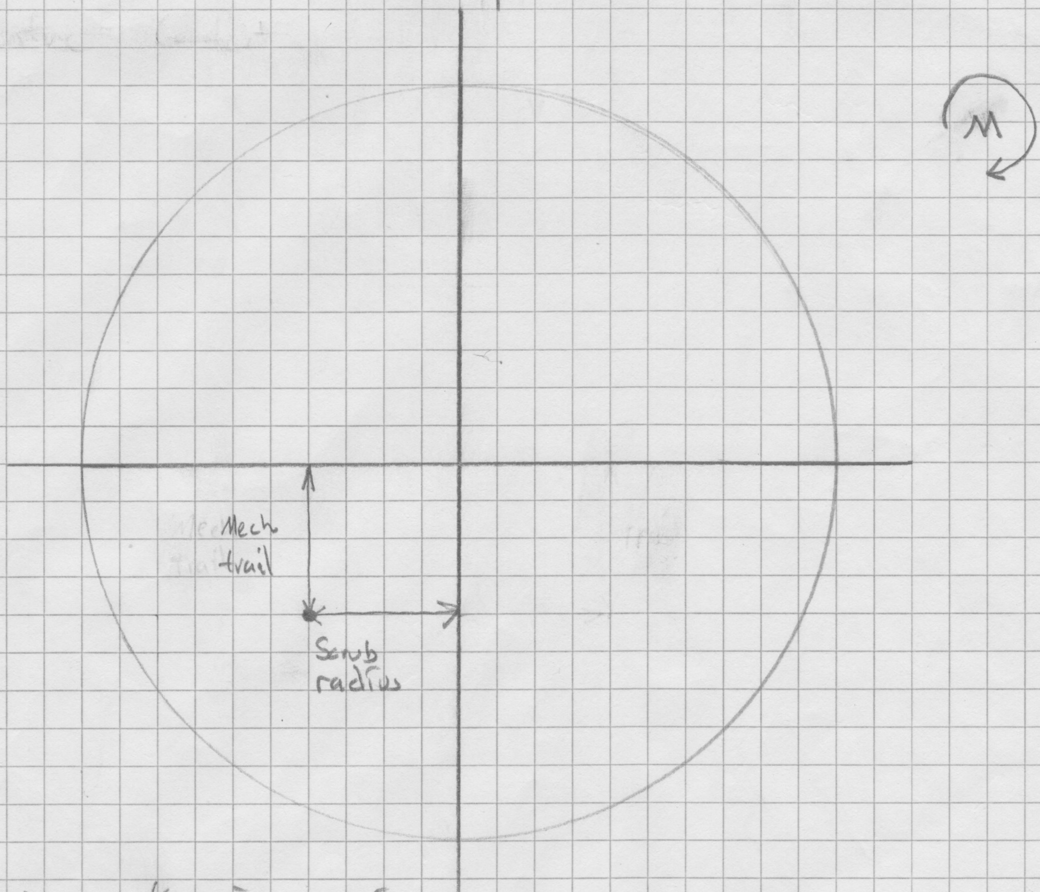
Braking forces on tire

$$[F_x, F_y, F_z] = [1, 0, 1]$$

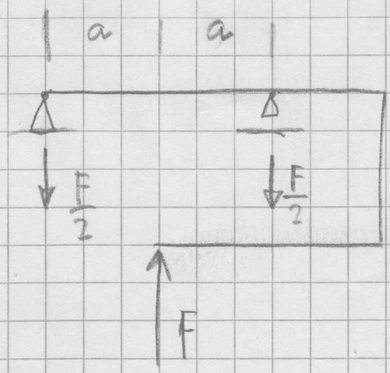


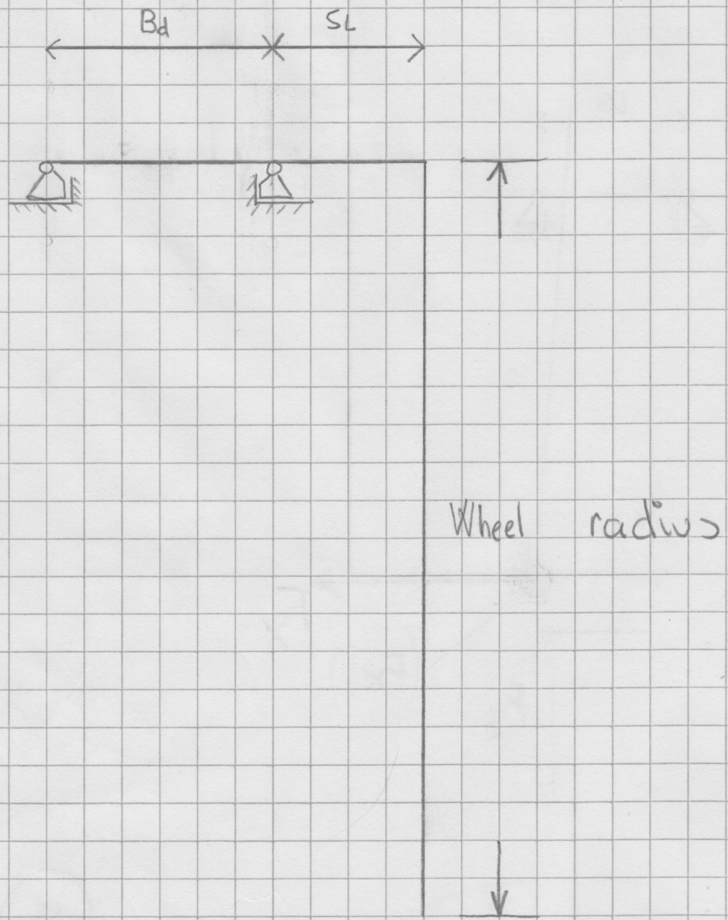
D_B = Bearing distance





Origin = Kingpin axis


a
a
F/2
F/2
F

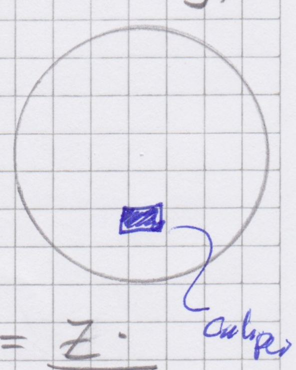

Bd
SL
Wheel radius

Problem 2

a)

Car weight = 300 kg approx

Total caliper weight = 2(0,29kg + 0,46 kg) including pads.
= 1,5 Kg



Calculating height of COG

$$\frac{1}{\text{Car weight}} \sum m_{part} \times Z = Z \cdot \frac{1,5\,kg}{300\,kg} = \frac{Z}{200}$$

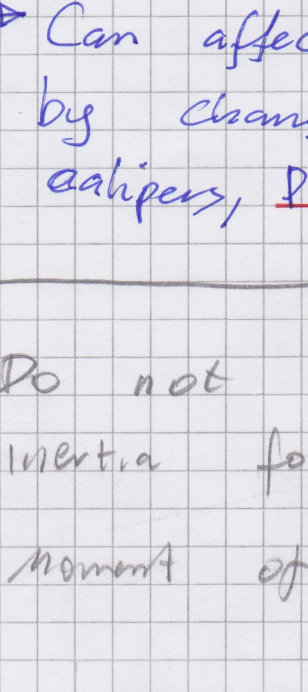
→ Can affect Centre of gravity 0,5% by changing the height of all brake calipers, Place calipers low.

Do not know Approximate Moment of inertia for the car

moment of inertia

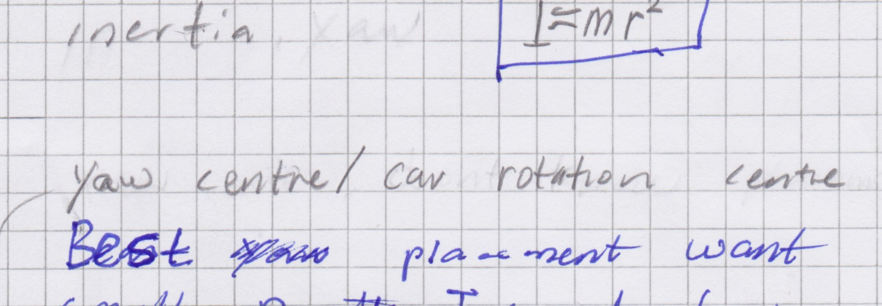
Squared reason for low r!

$$\boxed{I \approx mr^2}$$



Yaw centre/car rotation centre

Best ~~xxx~~ placement want small r. ~~Also~~ Inboard brakes is good!

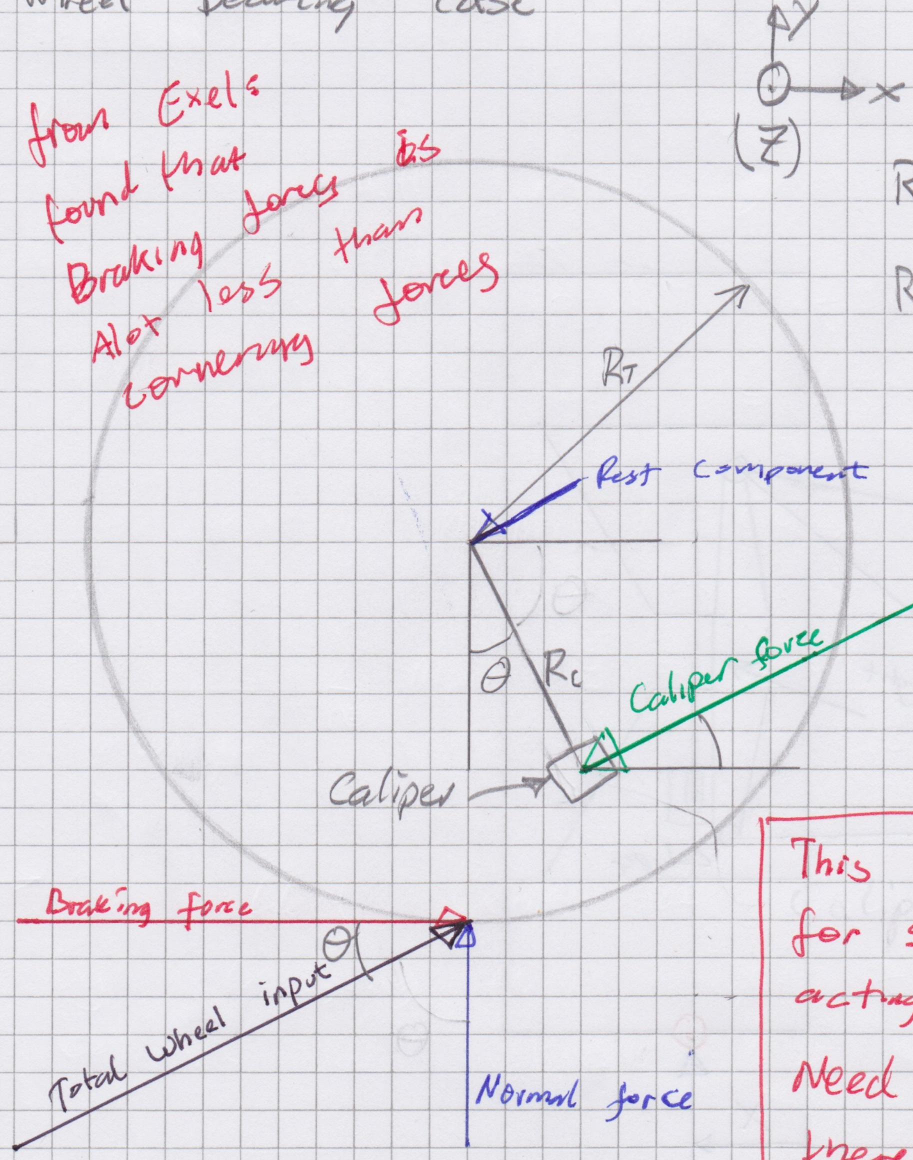


Wheel bearing case

from Exels found that is Braking force Alot less than cornering forces

R_T = Tire dia

R_C = Caliper dia



Caliper force Bigger than total wheel input

This evaluation for Same plane acting forces
Need to design thereafter

$$\text{Caliper force} = \text{Total wheel input} \times \frac{R_T}{R_C} \quad (R_T > R_C)$$

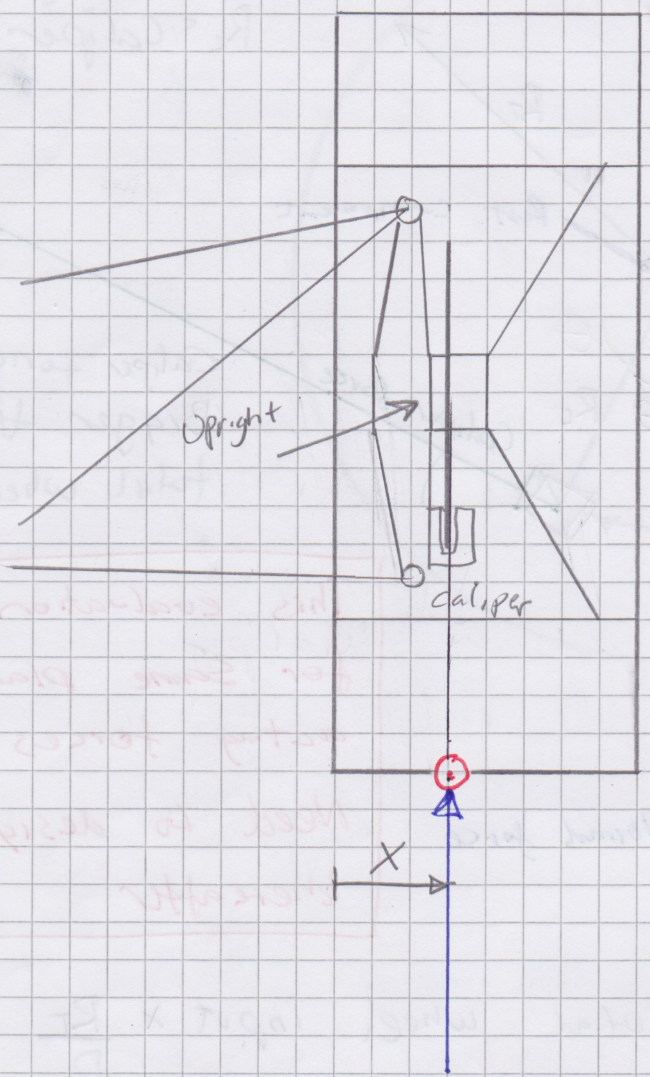
Best caliper placement radially θ

$$\arctan\left(\frac{\text{Normal force}}{\text{Braking force}}\right) = \arctan\left(\frac{N}{2N}\right) = 26{,}6^\circ$$

Want Caliper force direction and Wheel total force input to be in line/paralell Reacting eachother directly.

Angle of Wheel input force = $\underline{\underline{26{,}6^\circ}}$ →

θ of caliper should be the same



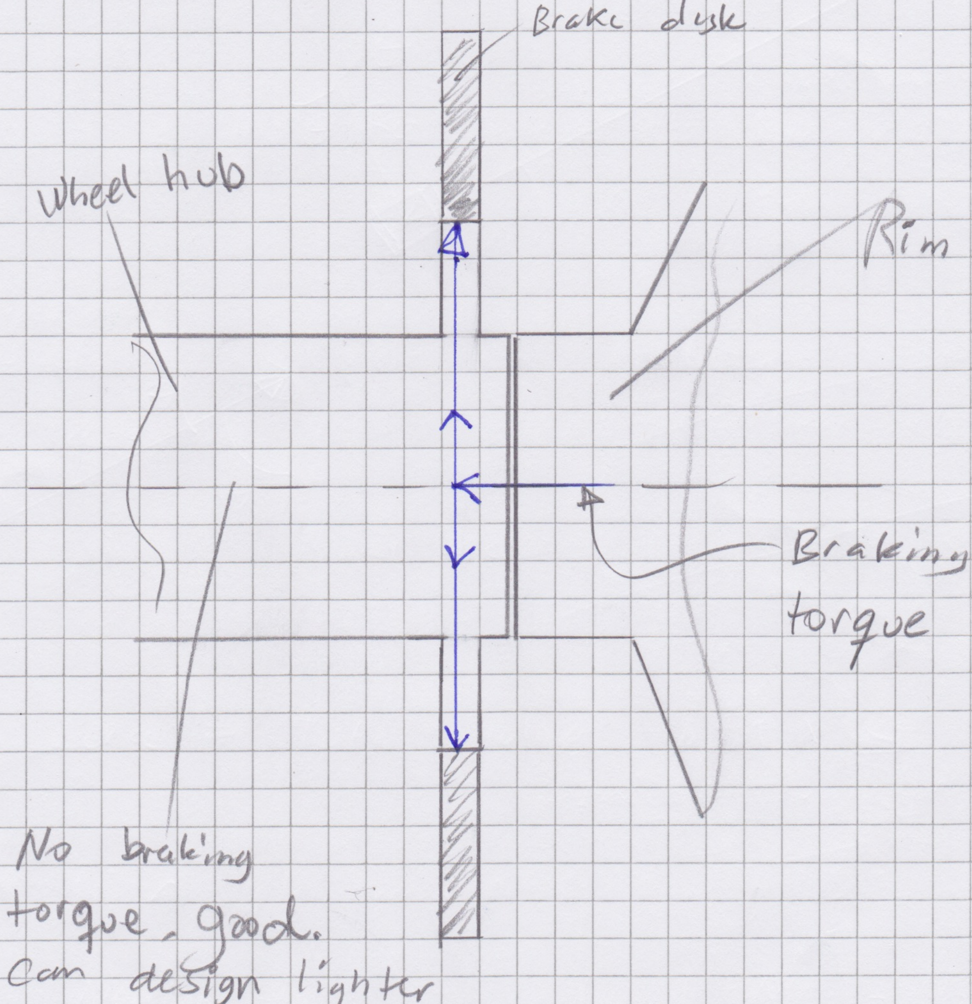
Find appropriate X, Brakedisk same placement (in line) for cancelling forces

Will have a rest from caliper force because it is bigger than wheel input forces

Simplicity of system and load paths. 1:23

- Want wheel braking torque to travel a least amount of distance from rim to Brakedisk

→ Inboard brakes bad! → Heavier axles etc



Braking torque, how bad really?

Hollow hub tackles torque good!

$$I_p = \frac{\pi\left((\text{outer diameter})^4 - (\text{inner diameter})^4\right)}{2}$$

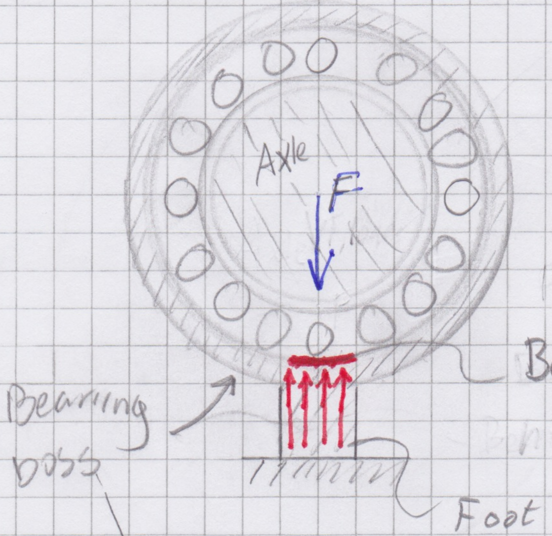
Polar moment of inertia

Problem 3 a

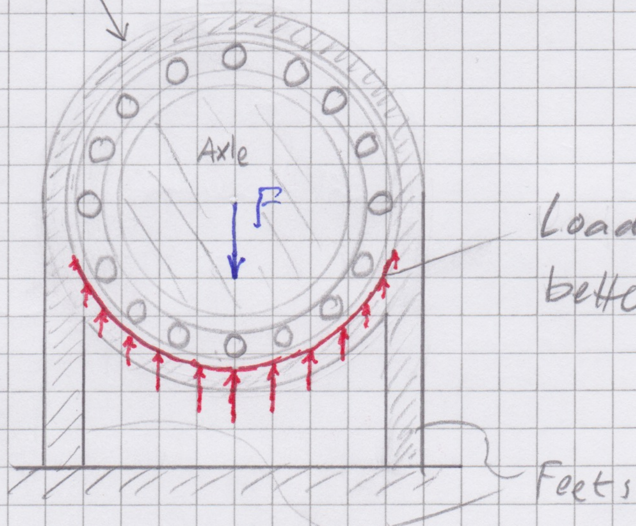
0,1146 0,1146 0,1588

0,0688

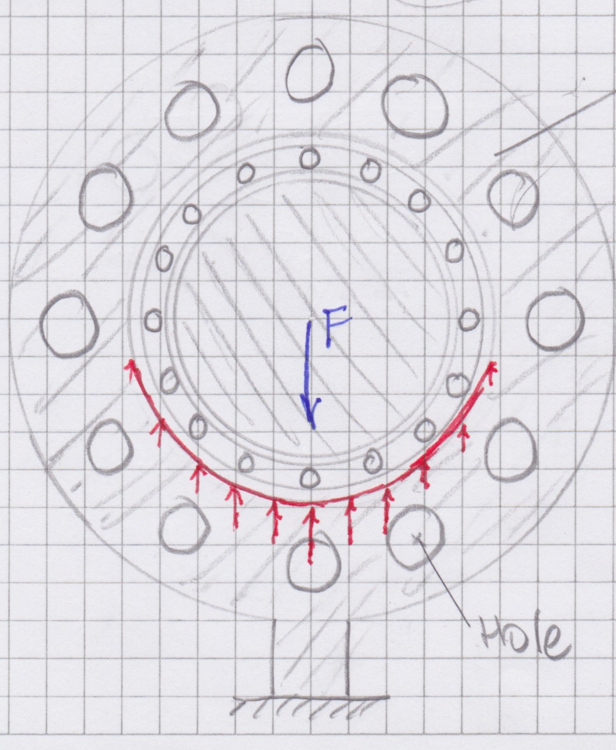
a)



Bearing only pointly loaded



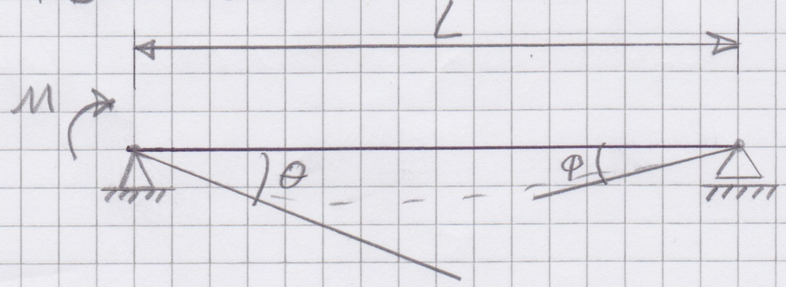
Load is distributed better



Thicker bearing boss with lightening holes to Utilize 2. Momentum of area. Create a Stiff but light ring Distributes forces better into bearing

Problem 4

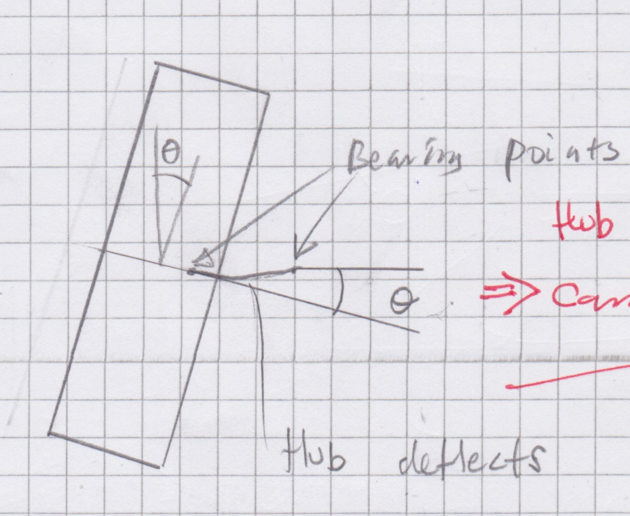
a). Hub model



Hub momentum input: M (Aligning, overturning)

$$\theta = \frac{M \cdot L}{3EI} \qquad \varphi = \frac{M \cdot L}{6EI}$$

Considering constant 2. area of momentum
I. want small bearing distance



Hub deflection
⇒ Camber deflection

Hub end angle deflection leads to camber compliance.

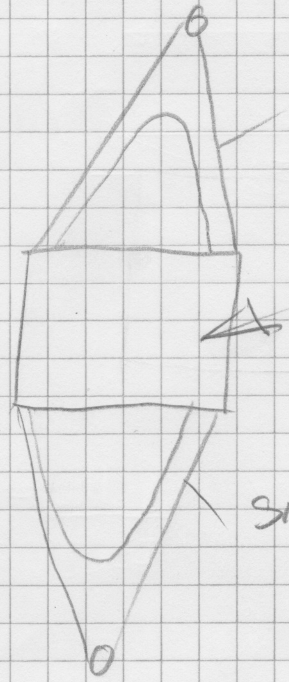
~~Am~~

Reason to choose hollow big hub

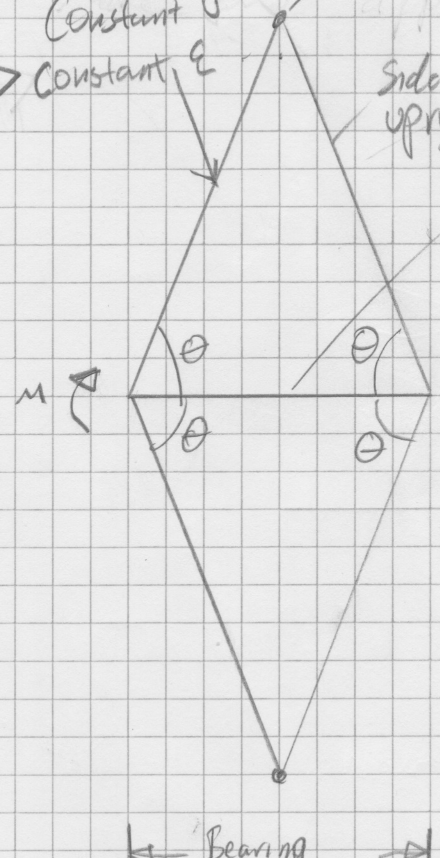
$$I = \frac{\pi \left(\text{Outerdia}^4 - \text{innerdia}^4\right)}{4}$$

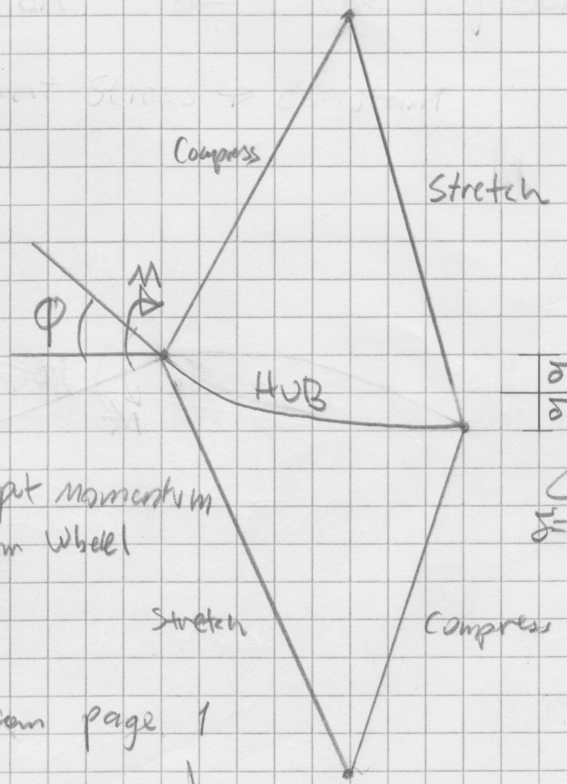
$\boxed{\text{4 th powers of strength !}}$

Design concept of upright is Constant Stress through the Beams



Suspension arm mounting



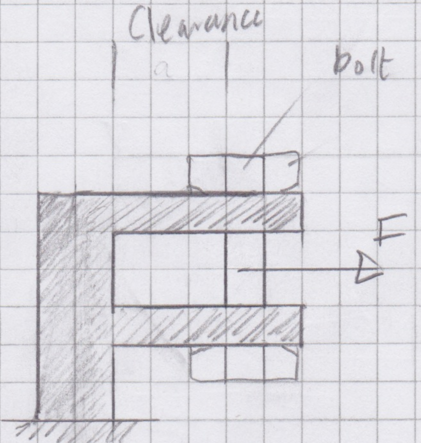


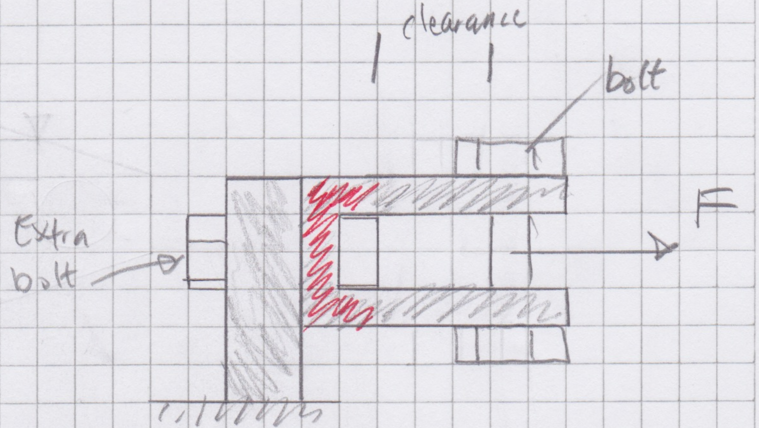
from page 1

$$\varphi = \theta + \frac{2\delta}{\text{Bearing distance}}$$

Problem 3 c

Consider simple example of bracket

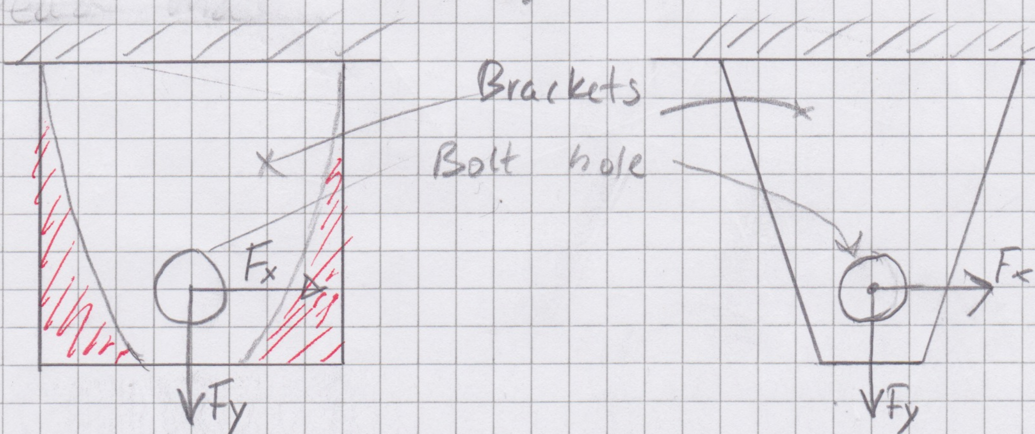




One piece
- Ridgid :)
- Simpler :) (often)
- If it breaks you are screwed :(

Bracket
- Extra bolt :(
- Extra Material :(
- Can replace :)

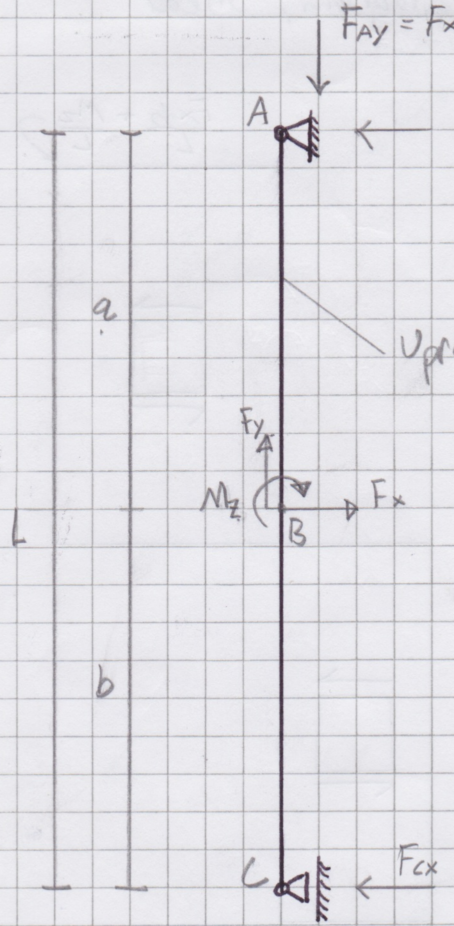
Dead material explained



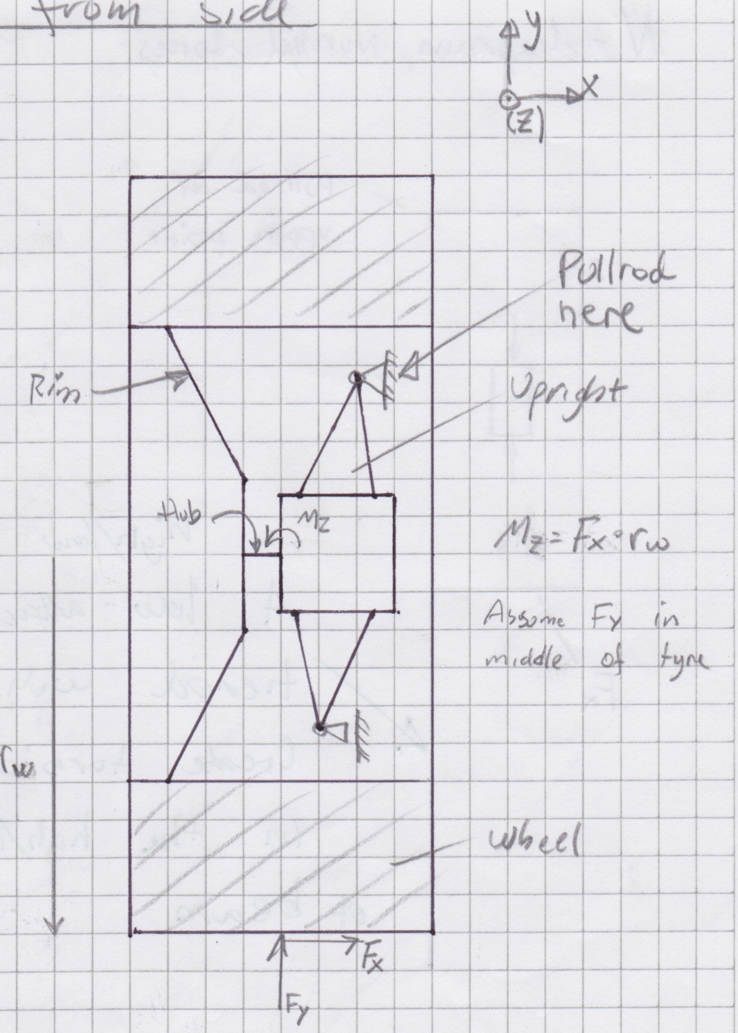
Red section is almost unused, thereby dead

lighter and better design less dead material

Upright forces seen from side



Sideview of upright



$M_z = F_x \cdot r_w$

Assume F_y in middle of tyre

$$F_{Ax} = \frac{-F_x \cdot b}{L} - \frac{M_z}{L}$$

$$F_{Ay} = -F_x$$

$$F_{Cx} = \frac{-F_x \cdot a}{L} + \frac{M_z}{L}$$

$$F_{Cy} = 0$$

$M_A = M_C = 0$ (Rodends)

F_x = Cornering force

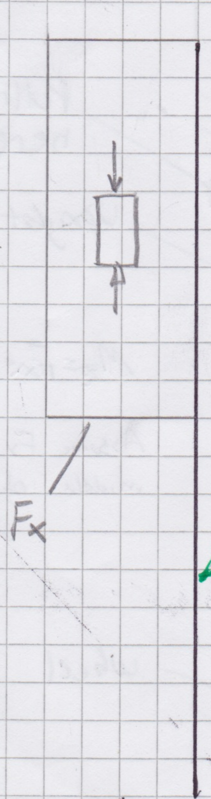
F_y = Tire load force

Assume:

$$F_x = \mu F_y \approx 2 F_y$$

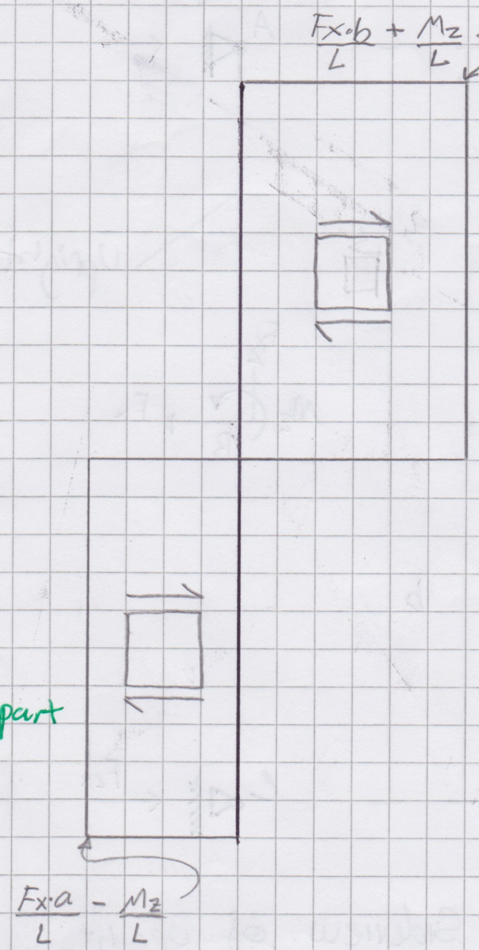
Tyre friction

N - diagram, Normal forces



high/low
A ~~low~~ - attached
tierod will
Create torsion
In the ~~high/low~~ part
of beam

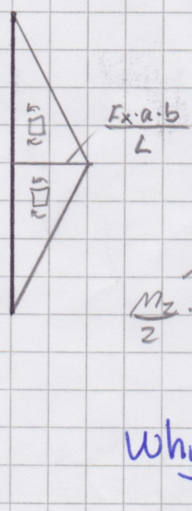
V - diagram, Shear

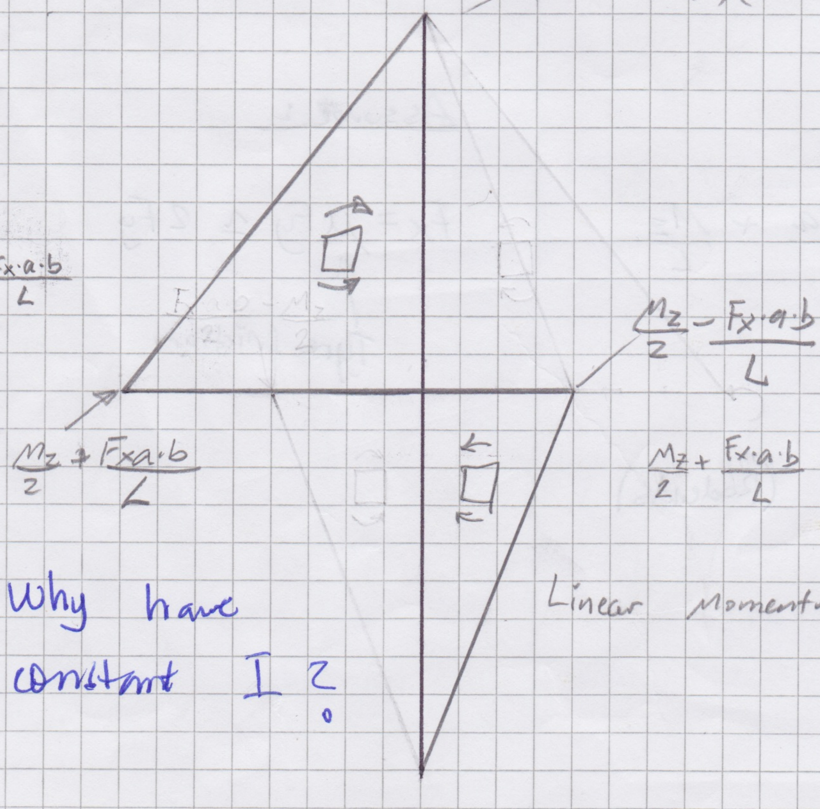


M - diagram, momentum

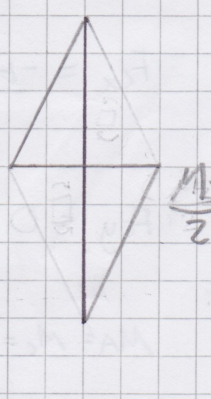
Free ends, (Rodend) $\Rightarrow M = 0$

From Fx





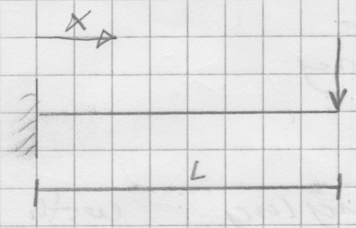
From Mz

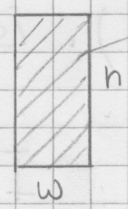


Linear Momentum diagram

Why have
constant I ?

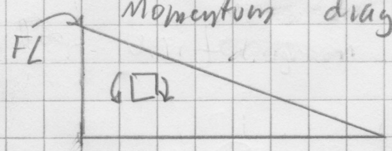
Beam strength, second Momentum of area I



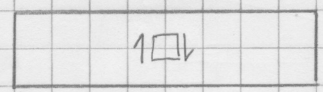


$$\sigma = \frac{\overset{M}{M \cdot h/2}}{I} + \frac{\overset{V}{F}}{w \cdot h}$$

Momentum diagram:



V diagram:



For regular nonchanging section beam:

$$I = \frac{\overset{w}{b} h^3}{12}$$

→ in the free end the beam is much stronger than needed. Only got shear here.

For beam with linearly changing width

$$W = \frac{b(x) h^3}{12}$$

Width is narrowing down till the end. $b(x)$ not 0 in end, got to deal with shear forces!

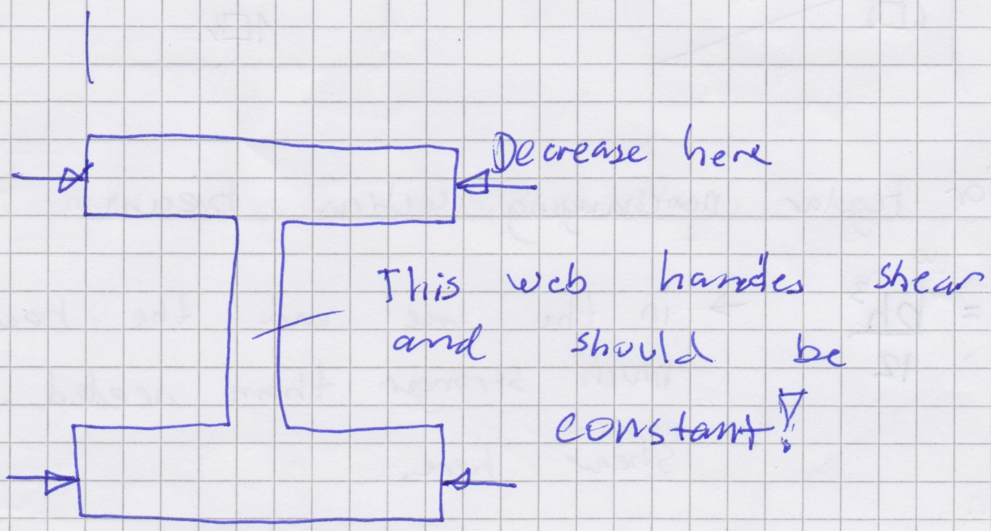
$$b(x) = ax + b$$

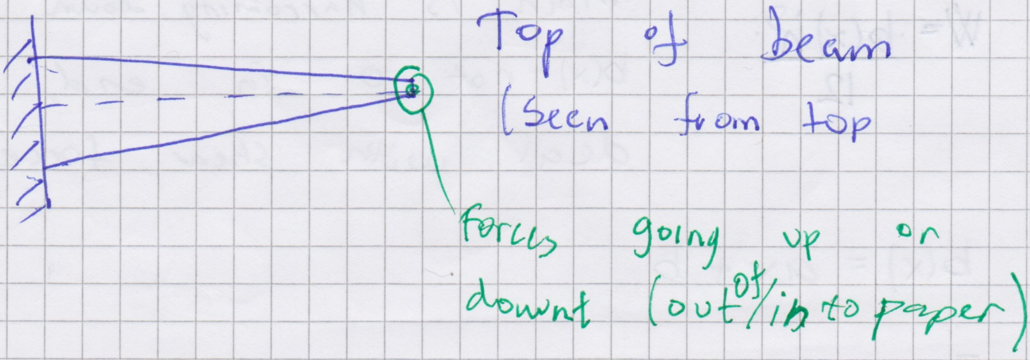
This beam will be lighter because it has only the strength needed at every point. the beam will have little more deflection compared to weight loss

Beam with constant I

→ K (curvature). is decreasing

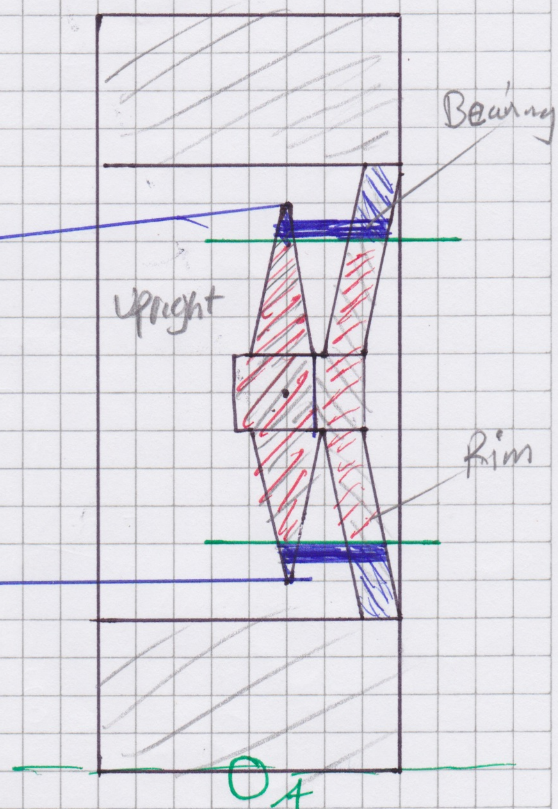
Can vary I by changing with of beam

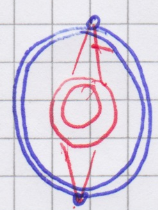




The bigger the bearing size and distance → The less Momentum forces the upright and hub system will have to deal with

Bearing/upright will have larger diameter (expands circularly)



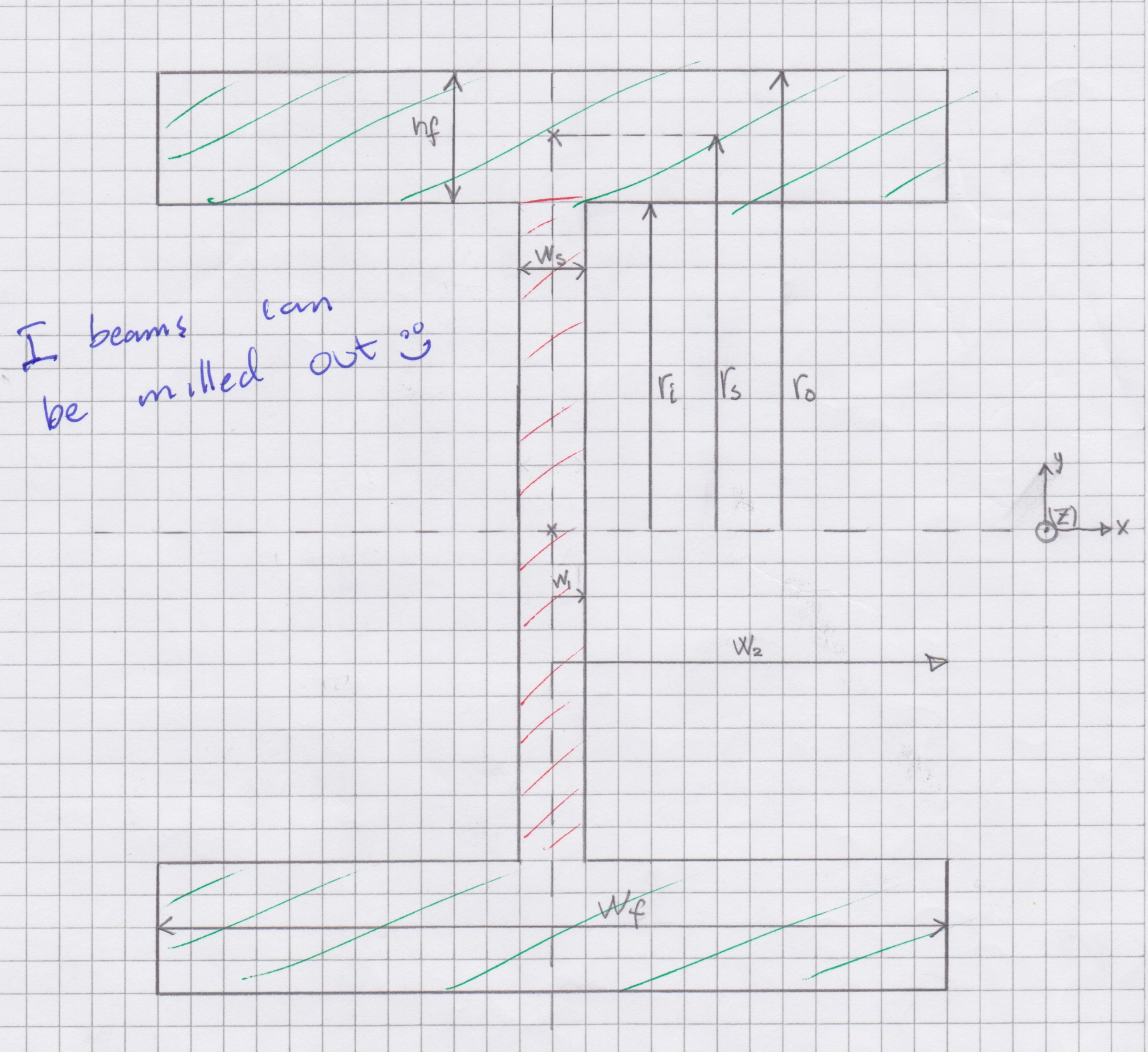


Upright 1
Upright 2

Can loose red stuff

~~The closer control arm is attached to A - line/Point the less Momentum's forces the upright and rim system will have to deal with~~

~~- The greater the bearing size, the less Momentum will the hub and upright system.~~

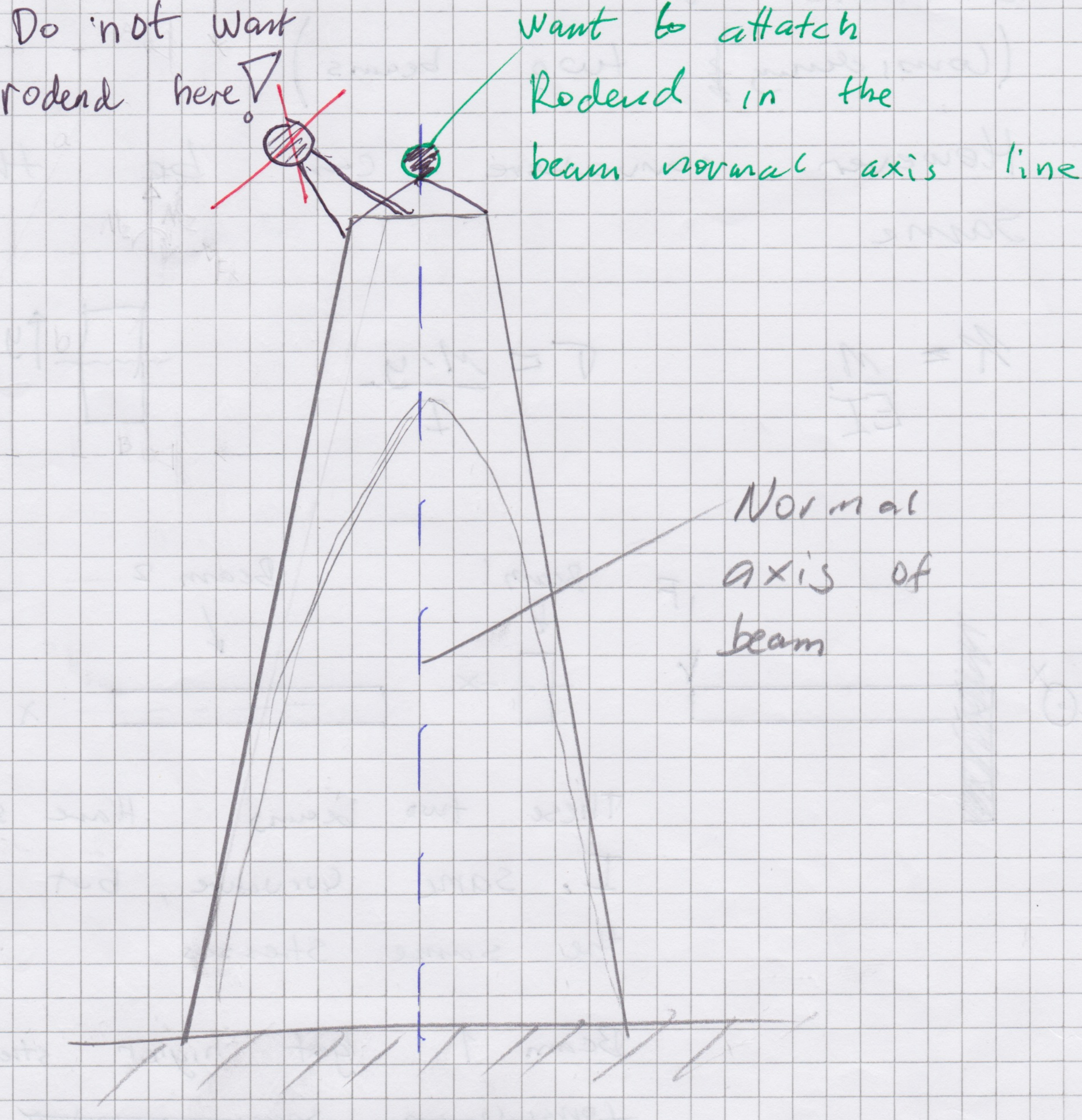


$$I = \frac{W_s \cdot r_i^3}{12} + \left(\frac{r_o + r_i}{2}\right)^2 \cdot W_f \cdot (r_o - r_i)$$

Red takes up shear and axis parallell shear

Green handles bending momentum

How to attatch to the beam

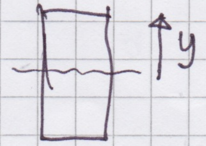


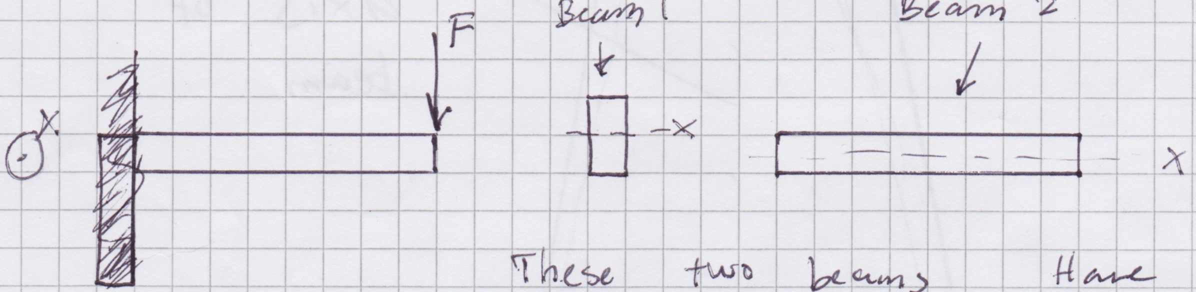
A rodent that is not attatched in the ~~line~~ Normal axis of the beam will create a torsional Component into the beam.

Constant I does not mean constant σ (considering two beams)

However curvature can be the same

$$\kappa = \frac{M}{EI} \qquad \sigma = \frac{M \cdot y}{I}$$





These two beams have same I, same curvature, but not the same stresses

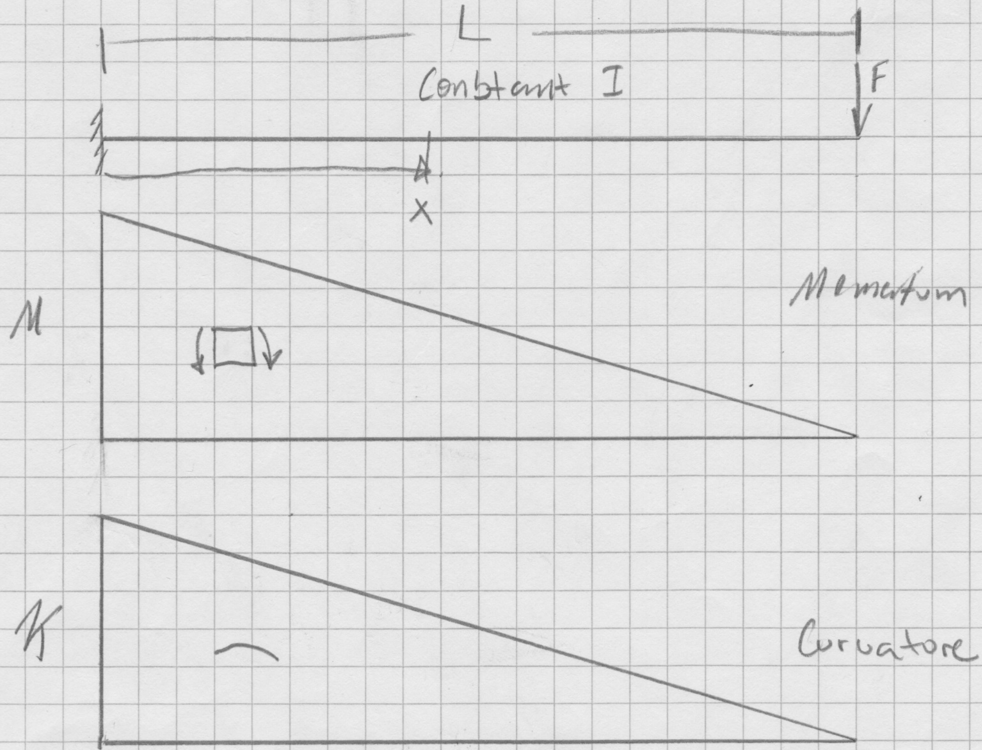
Beam 1 got higher stresses ~~considering same curvature~~

→ Want to have the tallest beam possible Build in height

~~Tall beams will handle~~

The same I does not mean same weight

Constant I beam with end load



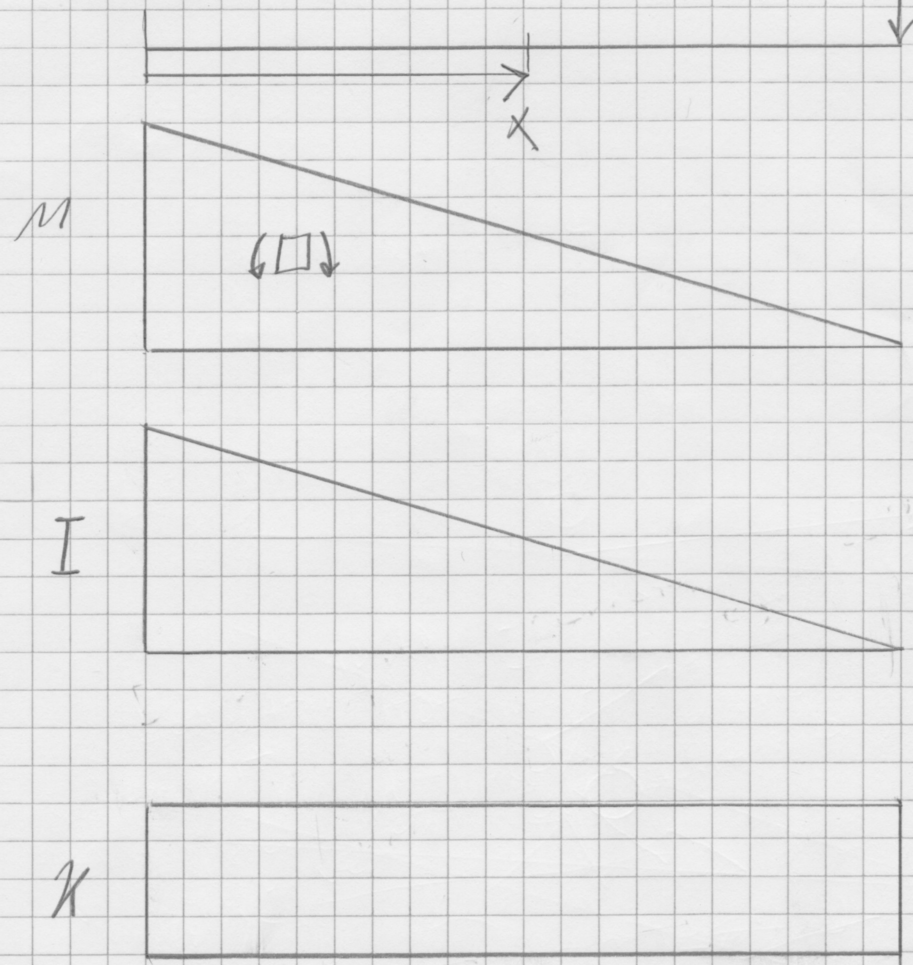
Angle of beam

$$\phi = K(0) - \int_0^L K \cdot x$$

displacement of beam

$$\int_0^L \phi \, dx$$

Linear I, Constant height beam Constant



$K = \frac{M}{EI}$

$K = \frac{F \cdot X}{I(x)}$

Constant curvature

$\frac{X}{I(x)} = \text{Constant}$

Angle of beam

$$\varphi = \int_0^{x} K \cdot dx = Kx$$

displacement of beam

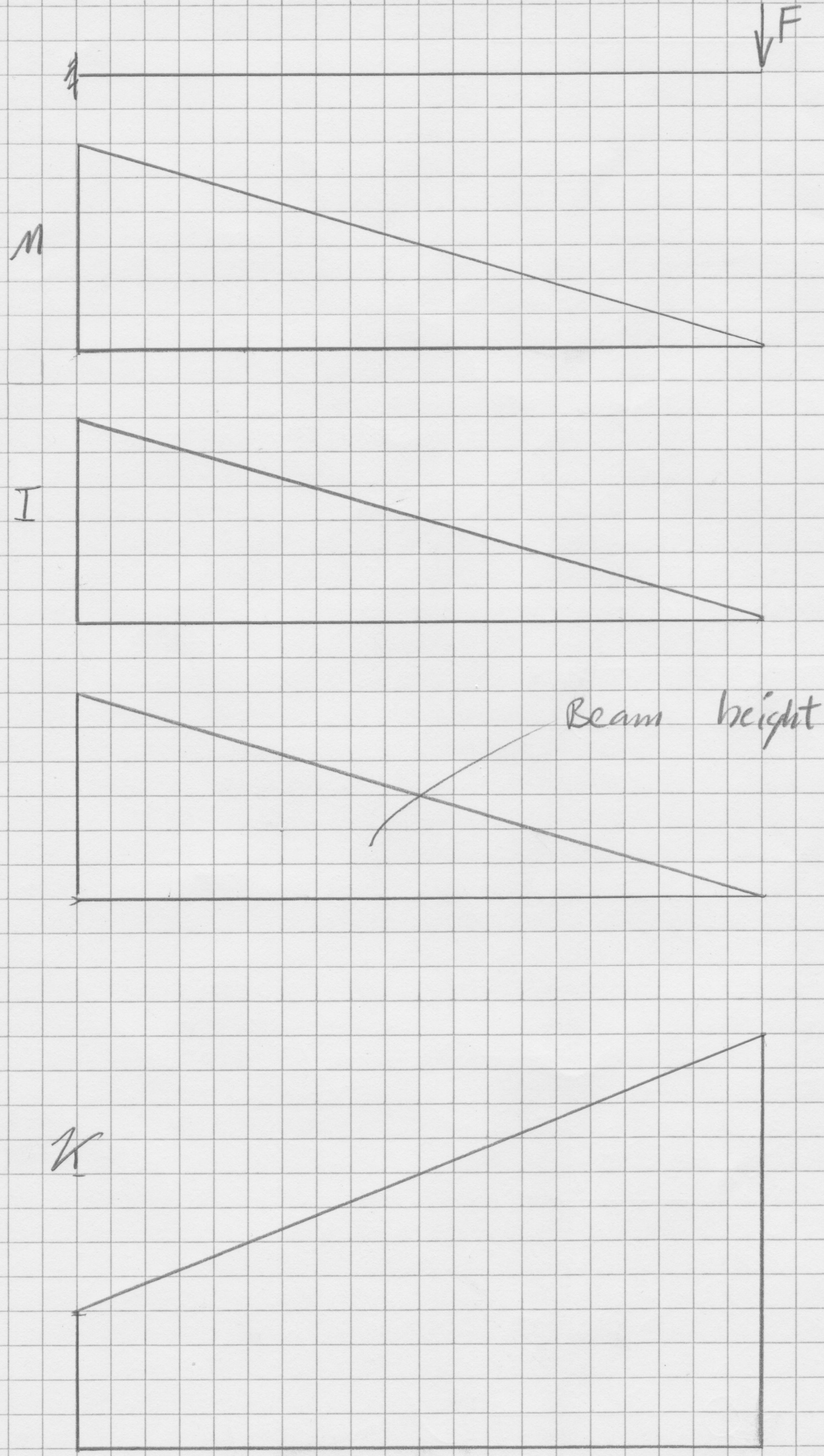
$$\int_0^{x} \varphi_x dx = \int_0^{x} Kx^2 dx = \frac{1}{3} Kx$$

In end $\frac{1}{3} KL^3$

$K = \frac{1}{\text{radius}}$

Beam deflects circulary

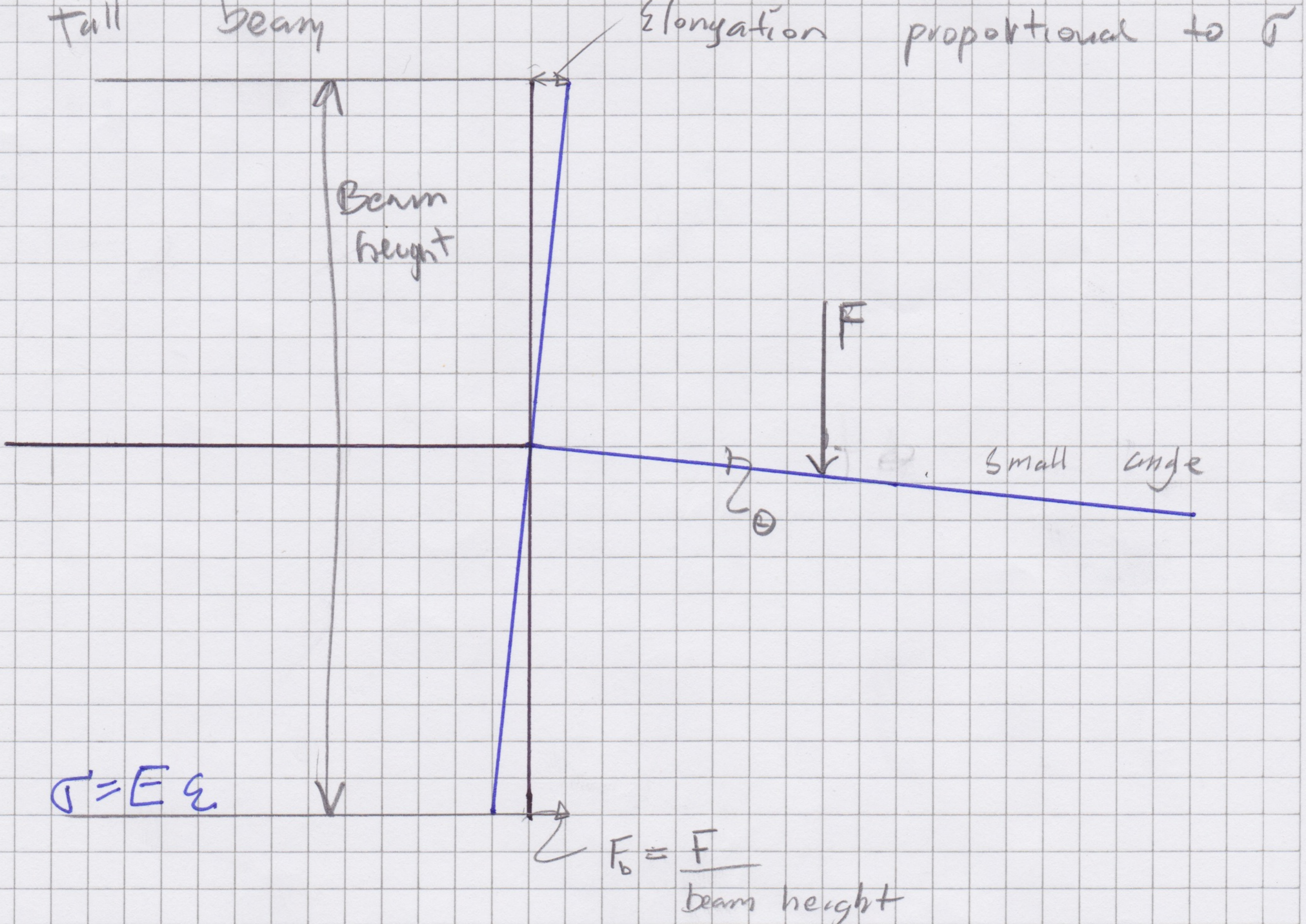
linear I, linear height beam

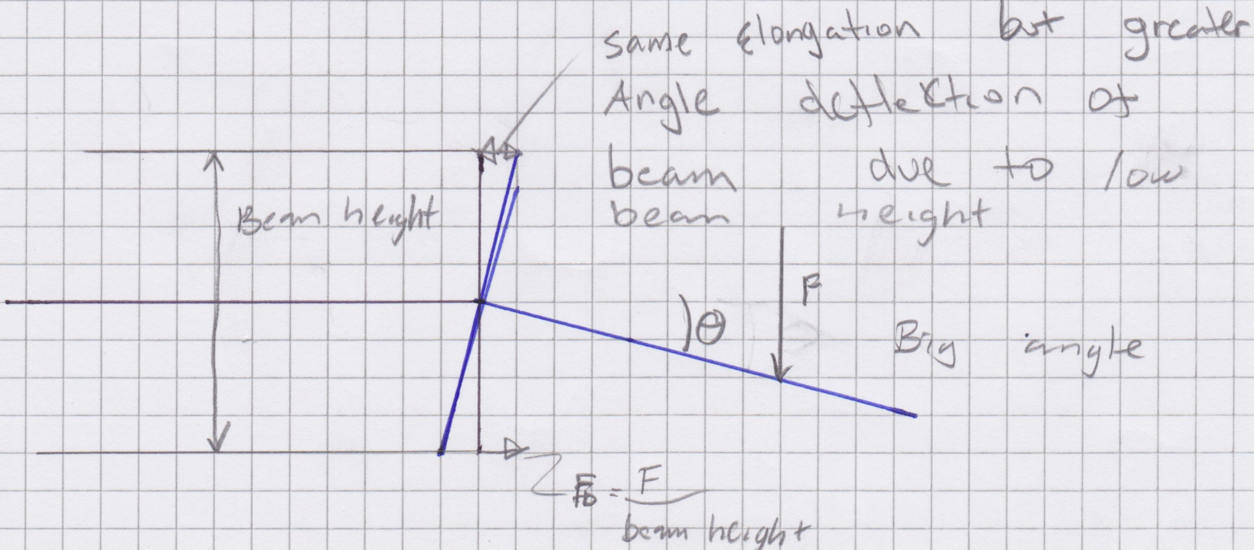


Increasing curvature, bad?

Constant stress Analysis

tall beam



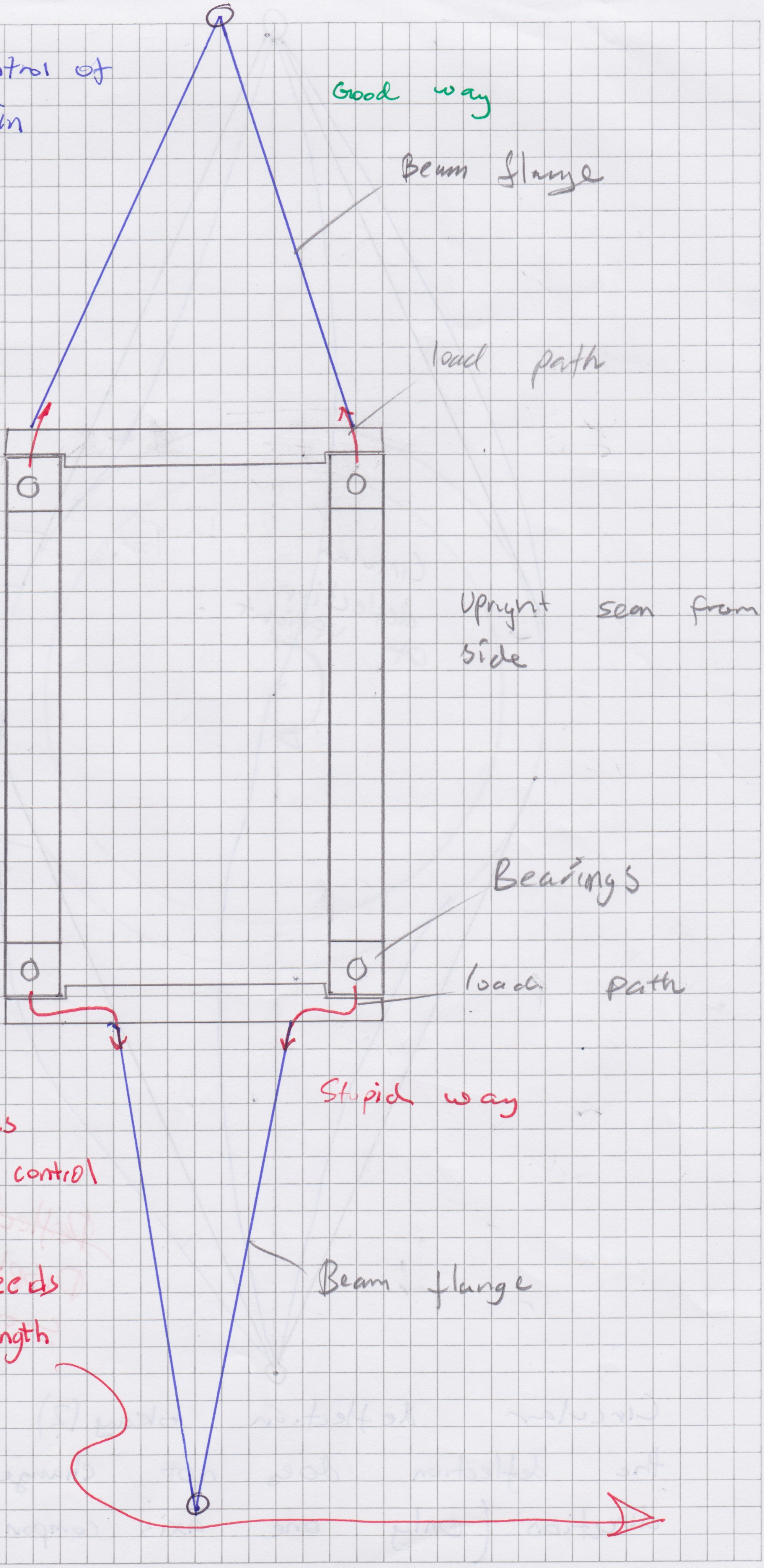


Want tall beams.

Constant stress beam does not have constant curvature

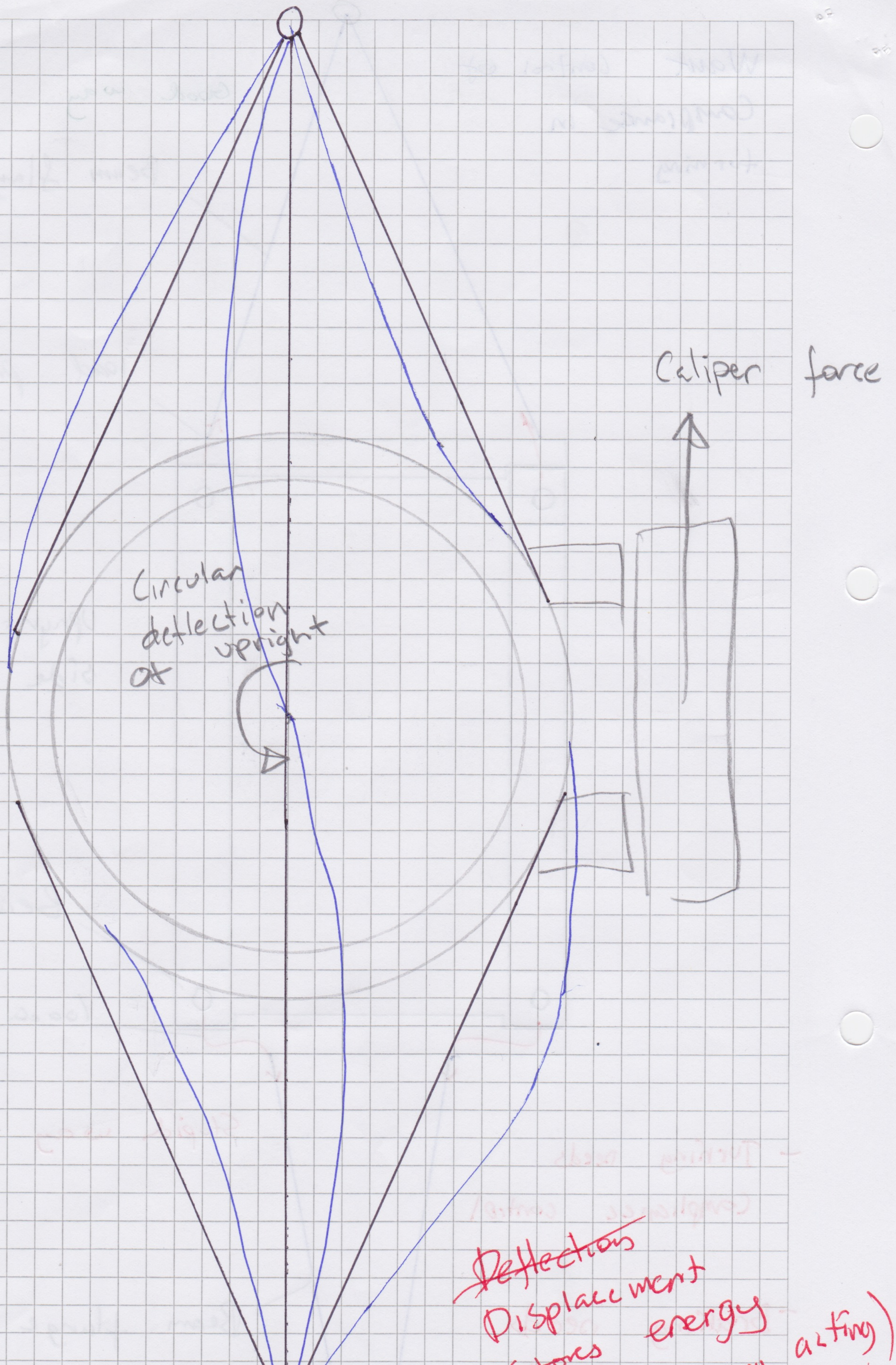
Only considering a beam that changes in height.

Want control of compliance in turning

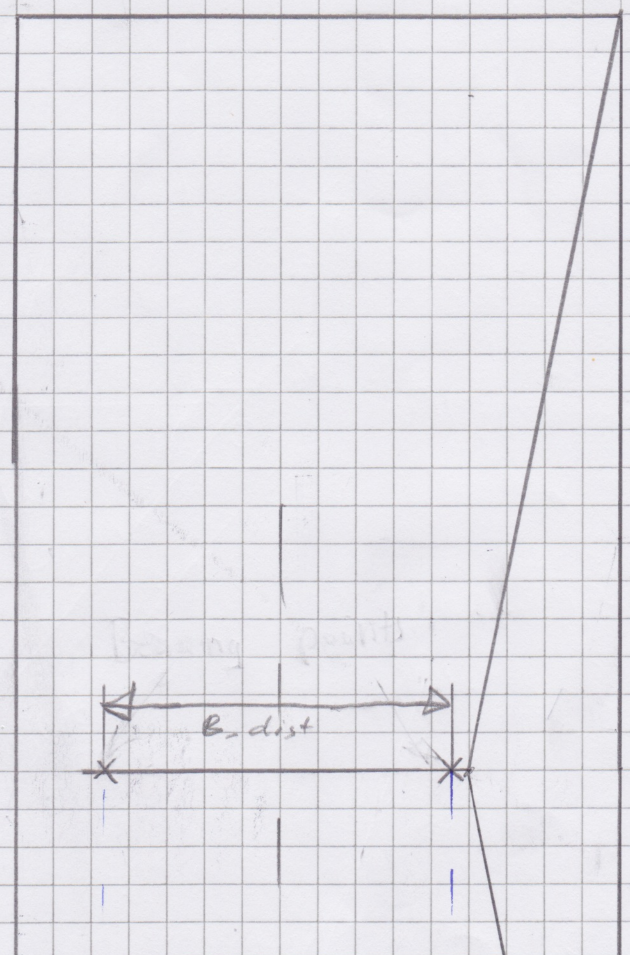


- Turning needs compliance control

- Braking needs only strength



Circular deflection okay (?) if the deflection does not change axis direction (only one axis component)



Want centre of forces inside bearing base distance

And visa versa

Next steps for reducing weight

- Reduce bearing distance, higher bearing forces
  Can iterate better with respect to wall thickness
  wich is now limited (2mm) beam is
  very low stressed

- Reduce bracket size

Upright too Stiff but can not decrease
thicknesses. Need to optimize basics.
Reduce bearing distance → Higher forces

Define beams mathematically, can analyse
deflection ~~??~~ and or Stress distribution
to weight